Mathematics 181 – D **Fall 2006**

Exam 1

September 19, 2006

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Name

Technology used:______________________ Directions:

- Be sure to include in-line citations every time you use technology.
- Include a careful sketch of any graph obtained by technology in solving a problem.
- **Only write on one side of each page.**

Do any six (6) of the following problems

1. (15 points) Use the **definition** of definite integrals as the limit of Riemann sums and the Useful Facts below to compute

$$\int_0^2 \left(12x^2 + 2x\right)\, dx.$$

[No credit for using the Fundamental Theorem of Calculus]

2. (15 points) Do any **two** (2) of the following

   (a) Use the definition (see Useful Facts below) to compute the discrete derivative of the following sequence $b(n) = (n+2)\,5^n$. (Use algebra to factor your answer.)

   (b) Explain why

$$\sum_{k=1}^{n} \left(k^7 + 2k\right) = \sum_{j=4}^{n+3} \left((j-3)^7 + 2j - 6\right)$$

   (c) Express the following limit as a definite integral where P is a partition of the interval $\left[0, \frac{\pi}{3}\right]$

$$\lim_{\|P\|\to 0} \sum_{k=1}^{n} \left(\tan(c_k)\ \Delta x_k\right)$$

3. (8, 7 points) Evaluate the following indefinite integrals.

   (a)

$$\int \left(2e^x + \frac{3}{x} + 4\sec^2(x) - 5\cos(x)\right) dx$$

   (b)

$$\int \frac{1}{u^4}\left(\frac{2}{u} - \frac{7}{u^3} + \sqrt[3]{u}\right) du$$

4. (5 points each) Do all of the following

   (a) What is the average value of the function $f(x) = x^5 - 7x^2 + 2$ on the interval $[2, 6]$? [**Do not** use a Riemann Sum]

(b) Given the function $f(x) = x^3 + 1$ with domain the interval $[0, 4]$. Write a Riemann sum for f using a partition P that divides $[0, 4]$ into 3 subintervals and where $\|P\| = 2$. Be sure to specify P as well as writing out the three terms in the Riemann Sum.

(c) Suppose that f and g are integrable functions and that $\int_a^b (2f(x) + g(x))\, dx = 5$ and $\int_a^b (f(x) - g(x))\, dx = 7$. Use properties of definite integrals to find $\int_a^b f(x)\, dx$ and $\int_a^b g(x)\, dx$. Show your work.

5. (8, 7 points) Do both of the following

(a) Find the derivative of

$$y = \int_{e^x}^{2} \tan^2(t)\, dt$$

(b) Find the derivative of

$$y = \int_{x}^{x^2} \frac{1}{t}\, dt$$

6. (15 points) Use substitution to evaluate any **two** (2) of the following indefinite integrals

(a)

$$\int \frac{1}{\theta^2} \sin\left(\frac{1}{\theta}\right) \cos\left(\frac{1}{\theta}\right) d\theta$$

(b)

$$\int \frac{\left(\sin^{-1} x\right)^2}{\sqrt{1 - x^2}}\, dx$$

(c)

$$\int \frac{dx}{x\sqrt{x^4 - 1}}$$

7. (15 points) The following is a list of the first few terms of a sequence $a(n)$ with domain $n = 0, 1, 2, \cdots$. Determine the formula for $a(n)$.

$$2, 1, 6, 17, 34, 57, 86, 121, 162, 209, 262, \cdots$$

[Hint: If $b(n)$ has terms $2, 5, 8, 11, 14, 17, 20, \cdots$, then the first few terms of the discrete derivative of $b(n)$ would be $(5-2), (8-5), (11-8), (14-11), (17-14), (20-17), \cdots$.But this is easily seen to be $3, 3, 3, 3, 3, 3, \cdots$. So Hence $D_n[b(n)] = c(n) = 3$.]

Useful Facts

1. •

$$\sum_{k=1}^{n} 1 = n \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

- $D_n[a(n)] = a(n+1) - a(n)$
- $n^{\underline{p}} = n(n-1)(n-2)\cdots(n-p+1)$
- $D_n[n^{\underline{p}}] = pn^{\underline{p-1}}$ and If $a(n) = n^{\underline{p}}$, then $A(n) = \frac{1}{p+1} n^{\underline{p+1}} + C$
- $D_n[r^n] = (r-1)r^n$ and if $a(n) = r^n$ then $A(n) = \frac{1}{r-1} r^n + C$
- $\sum_{k=m}^{n} a(k) = A(n+1) - A(m)$