Mathematics 181 – D **Fall 2006**

Exam 2

October 3, 2006

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Name

Technology used: ____ Directions:

- Be sure to include in-line citations every time you use technology.
- Include a careful sketch of any graph obtained by technology in solving a problem.
- **Only write on one side of each page.**
- **When given a choice, specify which problem(s) you wish graded.**

The Problems

1. (15 points) Do **one** (1) of the following.

   (a) Find the area of the region bounded by the graphs of $x = y^2$ and $x = -2y^2 + 3$.

   (b) Find the area of the region in the first quadrant enclosed by the curves $y = \cos\left(\frac{\pi x}{2}\right)$ and $y = 1 - x^2$.

2. (15 points) Do **one** (1) of the following.

   (a) Evaluate
   $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2\cos(\theta)\, d\theta}{1 + (\sin(\theta))^2}$$

   (b) Solve the initial value problem $\frac{ds}{dt} = 8\sin^2\left(t + \frac{\pi}{12}\right)$, $s(0) = 8$.

3. (15 points) The base of a solid is the region in the xy -plane bounded by the graphs of the parabolas $y = 2x^2$ and $y = 5 - 3x^2$. Find the volume of the solid given that cross sections perpendicular to the x -axis are squares.

4. (15 points) Do both of the following. Use the Method of Slicing on one and the Method of Cylindrical Shells on the other.

   (a) Set up, but **do not evaluate** a definite integral for the volume of the solid obtained when the region bounded by the graphs of the curves $y = \sqrt{2x}$ and $y = x$ is rotated about the line $y = -1$.

   (b) Set up, but **do not evaluate** a definite integral for the volume of the solid obtained when the region bounded by the graphs of the curves $y = \sqrt{2x}$ and $y = x$ is rotated about the line $x = -1$.

5. (15 points) Find the total length of the graph of $f(x) = 1/3x^{3/2} - x^{1/2}$ from $x = 1$, to $x = 4$. [Hint: Δs is a perfect square.]

6. (10 points each) Do any **two** of the following.

(a) Suppose that $F(x)$ is an antiderivative of $f(x) = \frac{\sin(x)}{x}$, $x > 0$. Express

$$\int_1^3 \frac{\sin(2x)}{x}\,dx$$

in terms of F.

(b) The disk enclosed by the circle $x^2 + y^2 = 4$ is revolved about the y - axis to generate a solid ball. A hole of diameter 2 (radius 1) is then bored through the ball along the y -axis. Set up, but do not evaluate, definite integral(s) that give the remaining volume of this “cored” solid ball.

(c) A solid is generated by rotating about the x - axis the region in the first quadrant between the the x - axis and the curve $y = f(x)$. The function f has the property that the volume, $V(x)$, generated by the part of the region above the interval $[0, x]$ is x^2 for every $x > 0$.Find the function $f(x)$.

(d) Find the volume of the following “twisted solid”. A square of side length s lies in a plane perpendicular to line L. One vertex of the square lies on L. As this vertex moves a distance h along L, the square turns one revolution about L. Find the volume of the solid generated by this motion. Briefly explain your answer.

(e) A solid sphere of radius R centered at the origin can be thought of as a nested collection of thin spherical shells.

   i. Set up a Riemann sum approximating the volume of this solid sphere by adding up the volumes of the thin, nested spherical shells. [Use the fact that a spherical shell of radius x has surface area of $4\pi x^2$.]

   ii. Write the definite integral that is equal to the limit (as $\|P\| \to 0$) of this Riemann Sum.

   iii. You may **not** use either the Method of Slicing or the Method of Cylindrical Shells.