

I. INTRODUCTION: WHAT DOES THE *APOLOGY* HAVE TO DO WITH MATHEMATICS?

Plato's *Apology* is interesting to scholars in all sorts of fields, studied as literature, philosophy, and history. It is a marvelously complex work, its range of interest reaching far beyond the world of "classics" and even—oddly enough—into the mathematical world. Anders Wedberg writes in *Plato's Philosophy of Mathematics*, "Plato and the Academy founded by him played a paramount role in the development of mathematics as a system of pure science... In his dialogues he also presented in outline a philosophy of mathematics which has proved to possess an astounding vitality" (Wedberg, 9). As one of the people who founded the mathematics, Plato certainly made use of it in his philosophical writings.

As a philosopher, Plato was interested in understanding the world, using different methods of finding truth. Harold Tarrant, an expert on Plato, points out that: "The true interpreter must somehow try to match Plato's tremendous breadth of interest as well as his philosophical depths. Ideally he will see himself as a philosopher, mathematician, historian, speech-maker, literary critic and as a moral and religious being" (The Last Days of Socrates, ix). The *Apology* is a work of philosophy, mathematics, history, oratory, literature, and religion; if any one of these aspects is ignored, it is impossible to understand the work as a whole.

To understand Plato's use of the axiomatic method in the *Apology*, one must consider the work in its mathematical context. If considered carefully, Socrates' use of back and forth questioning of Meletus in his defense at the trial turns out to be a mathematical proof.

II. PROOF IN THE *APOLOGY*: A BLEND OF MATHEMATICS AND PHILOSOPHY

However, before looking at particular examples of proof in the *Apology*, it would be helpful to look at some of the background of the *Apology*. The *Apology* was written by the Greek philosopher Plato (c. 427-347 B.C.) as a defense of Socrates, Plato's former mentor. Socrates was executed by the Athenians in 399 B.C. on charges of harming the state; it is this trial of Socrates which Plato is writing about in the *Apology*. While one cannot take the *Apology* as either an accurate rendition of either Plato or Socrates' philosophies in pure form, there seems to be a blend of the two. By using the style of Socratic questioning, Plato was able to make his arguments. Tarrant makes sense of this, writing that: "The medium of language (*logos*), and presumably of argument in particular, is thought to provide in a sense a reflection of the truth rather than a guarantee of it. The concept of formal validity is not yet in evidence, though already the connection between dialectical and mathematical procedure is present" (The Last Days of Socrates, xiii). Through his statements, Socrates in the *Apology* was using the concept of mathematical proof, though perhaps loosely.

The axiomatic system is a logical, systematic approach to understanding. Starting simply with certain axioms—truths that everyone would take for granted—Socrates could then create arguments or proofs, which leads to logical conclusions. In particular, Socrates does this through his characteristic questioning of his opponent to lead him in answering a series of questions, which would lead to the answer that Socrates was looking for. In an attempt to get his accuser Meletus to contradict himself, Socrates uses a method of proof known in the axiomatic system as a *Reductio ad absurdum* (RAA)

proof¹; his goal is to reduce Meletus' accusations into components that are inherently contradictory. Tarant explains this in slightly different terms:

Plato's principal tool of persuasion is of course argument. There are two particular terms which are often used in this context, elenchus² and dialectic. The former is Socrates' means of examining the soundness of the views of others. Typically an interlocutor will make a moral claim that Socrates cannot accept. He then secures the interlocutor's assent to some further proposition or group of propositions, and, accepting these, proceeds to demonstrate that they are inconsistent with the original claim. It is a tool for the exposure of problems with beliefs and inconsistencies in sets of beliefs rather than for demonstrating what is true and what is false.

Yet, Socrates' argument, unlike a mathematical argument, has a purpose; he is in a trial, defending himself. He is not simply using a proof just for the sake of deducing things from known axioms; he is using the method of proof to show his innocence of the charges. Wedburg contrasts these two elements:

The pure mathematician claims no truth at all for axioms and the theorems of the various geometrical systems he investigates. His only claim is that the theorems logically follow from the axioms... For Plato, the Euclidean geometry was not an abstract system, in the modern sense, but a doctrine which is either true or false, the notions of truth and falsehood being understood in an absolute sense (Wedburg 46,47).

The *Apology* contains more than just pure mathematics; as was noted from the start, it is a blend of disciplines. Socrates had a moral purpose in his work; as he tells the jury in his defense, he has been commanded to use this elenchus by the gods. He questions people in the agora in order that they might become wiser, aware of how little they actually know. In the same way, he is using elenchus in the court.

¹ Greenburg gives a definition of the RAA proof in *Euclidean and Non-Euclidean Geometries*: "In this type of proof you want to prove a conditional statement, H implies C, and you begin by assuming the contrary of the conclusion you seek... The RAA hypothesis is a temporary assumption from which we derive, by reasoning, an absurd statement ("absurd" in the sense that it denies something known to be valid)... Once it is shown that the negation of C leads to an absurdity, it follows that C must be valid" (42).

² Elenchus, as defined by Richard Robinson in *Plato's Earlier Dialectic*, "in the wider sense means examining a person with regard to a statement he has made, by putting to him questions calling for further statements, in the hope that they will deduce the meaning and the truth-value of his first statement. Most often the truth-value expected is falsehood; and so 'elenchus' in the narrower sense is a form of cross-examination or refutation" (7).

Socrates knows that the jurors know all about his teaching in the agora, that they have been taught since they were young that Socrates is dangerous. So, before he directly attempts to refute the accusations that he is being accused of, Socrates feels that he must first make some things clear. He begs the jury, “I ask you to accept my statement that my critics fall into two classes: on the one hand my immediate accusers, and on the other those earlier ones whom I have mentioned” (39). This is an axiom that the jury must believe in order to believe Socrates’ argument, that there are two sets of accusations that are different for which Socrates needs to defend himself. He needs to show this so he can first knock down the argument of the early accusers, and then move on to the argument against his later accusers.

III. SPECIFIC EXAMPLES OF PROOFS

While Socrates makes arguments against both the old and new, it is his dialogue with Meletus which contains the clearest proofs. When he has finished with the old accusers, Socrates turns to Meletus, his new accuser, and cross-examines him: he is setting up an RAA proof. The accusation that he is charged with is as follows: “Socrates is guilty of corrupting the minds of the young, and of believing in supernatural things of his own invention instead of the gods recognized by the State” (46). He approaches his defense in an orderly manner, saying “Such is the charge; let us examine its points one by one” (46).

First, while the affidavit claims that Socrates is corrupting the young, Socrates argues that instead, it is Meletus who is guilty—for Meletus is treating a serious accusation lightly. While Meletus claims to be concerned with Socrates corrupting the young, he, in fact, knows nothing about what is going on. In this section, Socrates makes

it clear what he is about to prove: “But I say, gentlemen, that Meletus is guilty of treating a serious manner with levity, since he summons people to stand their trial on frivolous grounds, and professes concern and keen anxiety on manners to which he has never given the slightest attention. I will try to prove this to your satisfaction.” (46).

A. PROOF #1: SOCRATES IS NOT GUILTY OF CORRUPTING THE YOUTH

1. “Come now, Meletus, tell me this. You regard it as supremely important, do you not, that our young people should be exposed to the best possible influence?” (46).

Meletus answers that he does think this.

2. Then, Socrates asks who the best influences are (and Meletus must know since he claims to be so concerned with this issue); Meletus pauses, unable to answer. Already, Socrates feels that he has proved a portion of his argument, that Meletus has not been paying attention. Just as in a proof where it is often necessary to prove different parts of the proof separately, Socrates breaks his argument into parts.

3. “Tell me, my friend, who is it that makes the young good?” (46).

Meletus answers, “The laws” (46). But this answer is not helpful to Socrates; it does not lead his argument in the direction he wants to go. So Socrates restates the question.

4. “That is not what I mean, my dear sir; I am asking you to name the person whose first business it is to know the laws” (46).

Meletus answers “These gentlemen here, Socrates, the members of the jury” (46).

5. Socrates gets Meletus to say that these men have the ability to make the young men better—all of the jurymen, not just one. Likewise, Meletus says that the spectators in the court, the members of the Council, and the whole population of Athens “has a

refining effect upon the young” (47). Meletus agrees that all of these people make the young better.

6. “Then it would seem that the whole population of Athens has a refining effect upon the young, except myself; and I alone corrupt them. Is that your meaning?” (47).

Meletus emphatically agrees to this.

a. Horse Training Lemma

Here, Socrates interjects an example as a way of proof. In a sense, this is similar to a lemma in a mathematical proof. Just as a lemma is a separate proof used inside a bigger argument, this “horse training lemma” is a vital part of proving Socrates’ argument that he is not corrupting the young. Socrates asks Meletus “Take the case of horses; do you believe that those who improve them make up the whole of mankind, and that there is only one person who has a bad effect on them? Or is the truth just the opposite, that the ability to improve them belongs to one person or to very few persons, who are horse-trainers, whereas most people, if they have to do with horses and make use of them, do them harm?” (47). It seems clear that the horse-trainers are the ones who make the horses better; the more people involved, the more the horses are harmed. Socrates says, “Is not this the case, Meletus, both with horses and with all other animals? Of course it is, whether you and Anytus deny it or not” (47). Socrates feels that he has proved this smaller point clearly. Now, he can use it to make his bigger point: that the same is true for people, that one person helps them, while the majority harms them. He has gotten Meletus’ statements to reach a contradiction, proving that Meletus’ statement is wrong because it can be reduced to an absurd statement.

7. Now, Socrates moves on. He begins by questioning Meletus whether it is better to live in a good or bad community? Of course, Meletus answers that it is better to live in a good community, and that the good people have a good effect while the bad people a bad effect. Then, Socrates asks if anyone prefers to be harmed rather than benefited by the people around him. Meletus answers, “Of course not.” (47). So Socrates asks, “Well, then, when you summon me before this court for corrupting the young and making their characters worse, do you mean that I do so intentionally or unintentionally?” (47). Meletus answers that Socrates is doing this intentionally. This statement leads to several possibilities, according to Socrates. Socrates argues that, if he is corrupting his associates, he runs the risk of them corrupting him; of course, no one wants this, so Socrates would not corrupt his friends intentionally. So, Socrates says, “Either I have not a bad influence, or it is unintentional; so in either case what you claim is false” (47). Socrates has proved Meletus’ argument wrong, once again, by getting him to make a contradiction.

He goes even further, using the same argument to turn the tables on Meletus, arguing that Meletus should be the one on trial. He says: “If I unintentionally have a bad influence, the correct procedure in cases of such involuntary misdemeanors is not to summon the culprit before this court, but to take him aside privately for instruction and reproof; because obviously if my eyes are opened, I shall stop doing what I do not intend to do. But you deliberately avoided my company in the past and refused to enlighten me, and now you bring me before this court, which is the place appointed for those who need punishment, not for those who need enlightenment.” (48).

Socrates feels he has not only proved that Meletus is wrong in the first part of the accusation but that it is Meletus who is, in fact guilty. Now, he can move on to the second part of the accusation: that he is guilty of believing in new deities instead of the gods of the state. His goal, once again, is to show that Meletus's formal charge contradicts itself.

B. PROOF #2: SOCRATES IS NOT GUILTY OF BELIEVING IN SUPERNATURAL THINGS OF HIS OWN INVENTION INSTEAD OF THE GODS RECOGNIZED BY THE STATE

Socrates attempts to define the charge that Meletus has brought against him: "Is it that I teach people to believe in some gods (which implies that I myself believe in gods, and am not a complete atheist, and so not guilty on that score), but in different gods from those recognized by the state, so that your accusation rests upon the fact that they are different? Or do you assert that I believe in no gods at all, and teach others to do the same?" (49). At least according to Socrates' beliefs—the things that he takes as foundational, the axioms in his system of argument—teaching people to believe in gods implies that he believes in gods. However, Meletus is charging him with disbelieving in the gods in total.

Meletus, here, gets some of his information wrong, claiming that Socrates says, "the sun is a stone and the moon a mass of earth," (49) clearly confusing him with Anaxagoras. Once again, Socrates shows that Meletus is not taking the charges seriously, not truly having considered them thoroughly.

So Meletus says, once again, that Socrates "believes in no god" (49). Socrates accuses Meletus of "devising a sort of riddle for me, saying to himself, 'Will the infallible Socrates realize that I am contradicting myself for my own amusement, or shall I succeed in deceiving him and the rest of my audience?'" It certainly seems to me that he

is contradicting himself in this indictment, which might just as well run: ‘Socrates is guilty of not believing in the gods, but believing in the gods.’ And this is pure flippancy”

(49). Socrates makes it clear here that he is defending himself with an argument of contradiction. The way that he is going to show this contradiction is by examining “the line of reasoning which leads me to this conclusion” (49).

1. “Is there anyone in the world, Meletus, who believes in human matters, and not in human beings?” (50).

- a. “Is there anyone who does not believe in horses, but believes in equine matters?” (50)

- b. “Is there anyone who does not believe in musicians, but believes in musical matters?”

The answer to a, b is clearly “no.”

- c. “Is there anyone who believes in supernatural matters and not in supernatural beings?”

Meletus says “no.” If Meletus believes in supernatural matters, then it follows that he also believes in supernatural beings.

2. “Well, do you assert that I believe and teach others to believe in supernatural matters?” (50).

According to Meletus’ earlier statement, the answer is “yes.”

3. “Do we not hold that supernatural beings are either gods or the children of gods?” (50).

Meletus says “Certainly” (50).

4. “Then, if I believe in supernatural beings, as you assert, if these supernatural beings are gods in any sense, we shall reach the conclusion which I just mentioned just now when I said that you were testing me with riddles for your own amusement, by stating first that I do not believe in gods, and then again that I do, since I believe in supernatural beings.” (50)

Socrates is saying that Meletus’ is contradicting himself, that Socrates, according to Meletus’ own words, believes in the gods. He continues on in the same direction, asking: “If on the other hand these supernatural beings are bastard children of the gods by nymphs or other mothers, as they are reputed to be, who in the world would believe in the children of gods and not in the gods themselves?” (50). Just as it would be ridiculous as to believe in the young of horses or donkeys and not in horses or donkeys themselves (50), it would be impossible for Socrates to believe in the things of the gods but not the gods. Socrates has finished his second proof.

IV. FLAWS IN THE PROOFS: WHAT DOES THIS MEAN?

Socrates, at least, seemed to find his own arguments convincing. He tells Meletus, as the conclusion of these arguments:

There is no avoiding the conclusion that you brought this charge against me to try me out, or else in despair of finding a genuine offence of which to accuse me. As for your prospect of convincing any living person with even a smattering of intelligence that belief in the supernatural does not imply belief in the divine, and again that non-belief in gods does not imply non-belief in supernatural beings and heroes, it is outside all the bounds of possibility.

And yet, it seems that his jury was not convinced; they find him guilty of the charges that Meletus has proposed. Does this mean that his arguments were flawed or that the jury was simply unwilling to be persuaded, no matter how good Socrates’ argument was? One must take into consideration both the argument as it stands as well as the jury’s potential predisposition to want to convict Socrates.

As the arguments are outlined above, it seems (at least at first) that all of his steps were justified; for Meletus agreed to each point. The first argument, that Socrates is not corrupting the minds of the youths, seems much more straightforward. However, the second proof seems more problematic. The proof that Socrates is trying to make is supposed to show that Socrates is not guilty of believing in supernatural things of his own invention instead of the gods recognized by the state, for that is what the affidavit is accusing him of. His argument ends up showing that by believing in any sort of supernatural beings, he believes in the gods. However, in step B1, there seems to be a flaw in his reasoning. Socrates asks if there is anyone who believes in human things, but not human beings, horses but not things pertaining to horses, musicians but not musical things. Of course no one does this. But then he twists the question the other way, asking if supernatural matters imply supernatural beings. Meletus says “no,” but only because he has in mind all of the things that Socrates has suggested before. It is not necessarily true that believing in supernatural matters implies supernatural beings. This single false step undermines the validity of the rest of his attempted proof. Instead, it appears at least, that Socrates is using this new method of logical argumentation is a deceptive way.

Whether Socrates has unintentionally made an error or he is using his pseudo-logical rhetoric to sound convincing does not matter to the jury. In either case, the jury would certainly have seen Socrates as attempting to trick them with his words, whether or not they were able to see the flaws in his argument. To the jury, Socrates was seen as simply twisting his words to make false things seem true, just as he had earlier been accused. In the beginning of his defense, Socrates attempts to address this problem:

What effect my accusers have had upon you, gentlemen, I do not know, but for my own part I was almost carried away by them; their arguments were so convincing. On the other hand, scarcely a word of what they said was true. I was especially astonished at one of their many

misrepresentations: the point where they told you that you must be careful not to let me deceive you, implying that I am a skilful speaker” (37).

Socrates claims that his only skill is in speaking the truth. And yet, from his own arguments, so carefully planned point by point to make a seemingly flawless argument, one can see why the jury believed that Socrates is doing exactly what his old accusers believed of him. Though Socrates begs of them to only “consider and concentrate your attention upon this one question, whether my claims are just or not” (38), the jury cannot trust him.

V. CONCLUSION

Did Socrates attempt to “fool” the jury? Or did he simply feel that his logical proofs were the best method of finding the truth, erroneously making a fatal flaw? This is not something that we fully answer in this analysis. There are too many other factors which, unfortunately, could not be discussed here. To fully understand the *Apology*, many different aspects must be discussed. Here, we have simply looked at it from a mathematical standpoint. And yet, one must not belittle the importance of mathematics in the *Apology*. To understand the argument that Socrates set forth in the *Apology*, one must have an understanding of the axiomatic principles behind it. As was inscribed on the entrance to Plato’s Academy: “Let no one ignorant of geometry enter this door” (Greenburg 1).

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