Problem 2.2

Algorithm to solve a Rubik's Cube

Chose a target edge cubelet, call it X.

Step 1—Place The Edge Cubelets in the UP Layer (ignoring orientation):

- If X is in the UP Layer.
    - If X is in the proper positions (orientation may be incorrect), chose a different X.
    - Apply F^2, R^2, B^2, or L^2 such that the move places X in the BOT Layer (this is the layer underneath the MID Layer).
    - Apply D until X is placed underneath its correct position.
    - Apply F^2, R^2, B^2, or L^2 to move X into its proper location
    - If not all UP edge cubelets are placed, chose another UP edge cubelet, call it X, and repeat step 1.
- If X is in the MID Layer (this is the layer underneath the UP Layer)
    - Apply U until an application of L, R, F or B permutes X while keeping the already placed edge cubelets in the UP layer. Call this process Z.
      NOTE: *this is possible since not all of the UP edge cubelets have been placed.*
    - Apply L, R, F or B such that it places X in the BOT Layer
    - Apply $Z^{(-1)}$
    - Apply D until X is placed underneath its correct position.
    - Apply F^2, R^2, B^2, or L^2 to move X into its proper location
    - If not all UP edge cubelets are placed, chose another UP edge cubelet, call it X, and repeat step 1.
- If X is in the BOT Layer (this is the layer underneath the MID Layer)
    - Apply D until X is placed underneath its correct position.
    - Apply F^2, R^2, B^2, or L^2 to move X into its proper location
    - If not all UP edge cubelets are placed, chose another UP edge cubelet, call it X, and repeat step 1.

Step 2—Orient the UP Edge Cubelets

While there is a UP Edge Cubelet with incorrect orientation
- Chose a UP edge cubelet with incorrect orientation, call it X.
- Reorient the cube such that X is facing you
- Apply $FR^{(-1)}D^{(-1)}RF^{(2)}$
- Continue if there is a UP Edge Cubelet with incorrect orientation

Step 3—Place UP corner cubelets with position and orientation
    Chose a target UP corner cubelet, call it X.

- If X is in the UP Layer
  - If X is in the correct position with proper orientation chose another X
  - Reorient the cube such that X is facing you and on the left
  - Apply LDL^(-1)
- X is now in the BOT Layer
  - Apply D until X is underneath its correct position
  - Apply D^2
  - Reorient the cube such that X is facing you and on the left
  - **NOTE**: *X is in the (f d l) position and its correct position is (b r u); i.e. they are "opposite"*
  - If the permutation (f d l, b r u)… places X correctly
    - Apply B^(-1)D^2B
    - Repeat step 1 if needed
  - If the permutation (f d l, u b r) … places X correctly
    - Apply RD^2R^(-1)
    - Repeat step 1 if needed
  - If the permutation (f d l, r u b) … places X correctly
    - Apply RD^2R^(-1)
    - **NOTE**: *X is in the correct spot, but wrong orientation*
    - Apply [B^(-1)D^2B][RD^2R^(-1)]
- Repeat Step 3 while more UP edge cubelets must be placed.

Step 4—Place MID Layer Edge Cubelets (ignoring orientation)

Chose a target edge cubelet, call it X

While there is a misplaced MID edge cubelet
- If X is not in the BOT Layer
  - If X is in the MID Layer and correct position chose another X
  - If X is in the MID Layer, but in the incorrect positions
    - Reorient the cube such that X is in the (f r) position
    - Apply L^(-1)D^(-1)(F^2D^2)^3DL
    - **NOTE:** *Now X is in the BOT Layer*
- Now X is in the BOT Layer
- Reorient the cube such that X's correct position is in the (f r) position
- Apply D until X is in the (d l) position
- Apply L^(-1)D^(-1)(F^2D^2)^3DL
- X is now in the correct position, repeat if needed; otherwise go to Step 5

Step 5—Orient MID Layer Edge Cubelets

Chose a MID layer edge cubelet with incorrect orientation and call it X

While there are incorrectly oriented MID edge cubelets
- Orient the cube such that X is in the (f r) position
- Apply L^(-1)D^(-1)(F^2D^2)^3DL

- Apply D^2
- Apply FD^(-1)F^(-1)D^(-1)FD^2F^(-1)DFDF^(-1)D^2
- Repeat if needed; otherwise go to Step 6

Step 6—Place BOT Layer Corner Cubelets (ignoring orientation)

Apply B until at least one BOT corner cubelet is in its correct position

While all the corner cubelets are not in their correct positions
- If there are three corner cubelets not positioned correctly
    - **NOTE**: *There is a pair of corner cubelets (that are in the same face) such that swapping those edge cubelets will correctly place one of them.*
    - Reorient the cube such that those corner cubelets are in the front face
    - Apply R^(-1)D^(-1)RFDF^(-1)R^(-1)DRD^2
    - **NOTE:** *Now there are only two misplaced corner cubelets*
    - *Repeat Step 6*
- If there are two corner cubelets not positioned
    - Chose a corner cubelet that has not been placed, call it X
    - Reorient the cube such that X is in the (f d l) position
    - If the other unplaced corner cubelet is in the (r b d) position
        - Reorient the cube such that X is in the (b l d) position
        - Apply R^(-1)D^(-1)RFDF^(-1)R^(-1)DRD^2
        - Reorient the cube such that X is in the (f d l) position
        - Apply R^(-1)D^(-1)RFDF^(-1)R^(-1)DRD^2
        - **NOTE***: Now there are two unplaced corner cubelets that are in the same face.*
        - Reorient the cube such that those corner cubelets are in the front face
        - Apply R^(-1)D^(-1)RFDF^(-1)R^(-1)DRD^2
        - Continue to step 7
    - Otherwise the two unplaced corner cubelets are in the same face
        - Reorient the cube where that face is the front face
        - Apply R^(-1)D^(-1)RFDF^(-1)R^(-1)DRD^2
        - Continue to step 7

Step 7—Orient the bottom corner cubelets

NOTE: *R^(-1)D^(-1)RD^(-1)R^(-1)D^2RD^2 fixes all corner cubelets and all UP edge cubelets while changing the orientation of the (f r d) , (r b d) and (l b d) corner cubelets*

- If none of the BOT corner cubelets have correct orientation
    - Apply R^(-1)D^(-1)RD^(-1)R^(-1)D^2RD^2 until at least one cubelet has proper orientation.
    - Reorient the cube such that the properly oriented corner cubelet is in the (f l d) position.

- o Continue applying R^(-1)D^(-1)RD^(-1)R^(-1)D^2RD^2 until all BOT corner cubelets have correct orientation.
- Otherwise reorient the cube such that one of the properly oriented BOT corner cubelets in the (f l d) position and apply R^(-1)D^(-1)RD^(-1)R^(-1)D^2RD^2 until all BOT corner cubelets have correct orientation.

Step 8—Place BOT Layer Edge Cubelets (with correct orientation)

Let Z = L^(-1)RF^(-1)LR^(-1)D^2L^(-1)RF^(-1)LR^(-1)
    Y = FLB^(-1)L^(-1)BF^(-1)DBD^(-1)B^(-1)

NOTE: *Z = (r d , l d , d b)*
        Y= (r d , b d , d l)

While the cube is not solved
- Choose a misplaced BOT Edge Cubelet
- Reorient the cube if necessary and apply Z or Y such that X is placed correctly with correct orientation
  NOTE: Do not simply place X choose Z or Y such that it places X with correct orientation.
- Repeat if necessary

End Algorithm