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Abstract Algebra
Problem 2.3

1. Find the position of the (1), fix that position as the beginning of the solution.
2. Find the (2), using rotations (**R**) and spins (**S**) where **R** is a clockwise rotation of the tiles along the track, $\mathbf{R}^{-1}$ is an anticlockwise rotation and **S** is a spin of the purple disk.
    a. The (2) will be either three disks away, or less than three disks away.
        i. If the (2) disk is three disks away from the (1) disk, with the (2) disk in the right side of the purple disk and do one more **S**.
        ii. If the (2) disk is two disks away from the (1) disk;
            1. Do a **S** with the (2) in the left side of the purple disk. The (2) will now be four disks away.
            2. Rotate the disks so that the (2) disk is second to the right on the purple disk, then do and **S** so that the (2) disk is now is three disks away from the (1) disk.
            3. Rotate the disks so that the (2) disk is on the right side of the purple disk. Now do an **S** so that the (2) disk is next to the (1).
        iii. If the (2) disk is a disk away from the (1) disk;
            1. Move the (2) disk to the left side of the purple disk and do an **S**, then the (2) will be seven disks away.
            2. Put the (2) disk in the second to the right spot on the purple disk and do a spin, this will put the (2) three away from the (1) disk.
            3. Another **S** with the (2) in the far right position will move this next to the (1).
        iv. If the (2) disk is next to the (1) disk than you are done.
    b. Do this for the other disks up to (16); these can be easily arranged using these steps.
3. Arrange the last four disks.
    a. There are 24 different possibilities for the ways in which the last four can be arranged. Twelve of these can be reached from an **S** from the other 12.
    b. To arrange the last four, move one disk at a time around the track until it is in the correct position. This can be done using the following process
        i. Put the tile you wish to move in the left side of the purple spin box.
        ii. Do $\mathbf{SR^{-1}SRSR^{-1}SR}$, this will move the tile onto the other side of the next three tiles.
        iii. Do $(\mathbf{R^{-1}})^{3}$ to put the tile back in the left side of the purple spin box.
        iv. Repeat this process until the tile reaches its correct spot
            1. Eventually the tile you are moving will reach every place in the track because three and 19 are co-prime.
        v. Do this for all the out of order disks.

4. Each different permutation of the tiles is a different permutation in S_{20}. Since each tile can be put in each position of the track, all possible permutations can be reached, so there are 20! possible positions.