The Axioms of Voice Leading:
A Musical Analysis

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Introduction:

Which came first, the axioms or the geometry? The answer is, of course, the geometry. Euclid's five essential axioms are descriptive of a mathematical system that has always existed as an innate component of the universe we inhabit. He did not invent geometry, he merely compiled and established a set of rules already obeyed by the system. These rules in no way influence or alter the system: they simply describe the behavior that is taking place. Without the axioms, geometry would still be just as real.

This same principle can be applied to countless disciplines. When you write a paper, for example, there are certain rules that you follow, even if only subconsciously. You open with an introduction and thesis statement, begin and end paragraphs where logical to the flow of ideas, and maintain a smooth structure until arriving at the conclusion. Now, it is not as if anyone "invented" the idea of writing a paper and then established a specific set of rules by which such composition must abide. People simply write in consequence of a desire and need to communicate, and organize their thoughts so as to be easily understood by the reader. Just as the audience of an author has certain innate expectations regarding what they want from a written work, so too does the audience of a composer entertain particular subconscious desires regarding what they expect to hear.

An elementary concept of music theory has been in existence since the society of ancient Greece, although this early theory was greatly intertmixed with alchemy and mysticism (Evans). It was many years before a strict study of the discipline was carried out, establishing music theory as a diverse and complex subject of investigation. General music theory, however, is a very

broad topic, just as math is a very broad topic, so I intend to focus solely on the establishment of the principles of voice leading.

Voice leading itself is an unavoidable aspect of the composition of music. In simple terms, it is the movement of the individual voices, or tones, of a chord as the harmonic structure changes from one chord to another. Here is an example (Kostka, 88):

In this—and any—example of four part choral writing the top line is referred to as the soprano, the second as the alto, the third as the tenor and the bottom, the bass. As a self-contained melody, each line moves from note to note in a smooth and sensible fashion.

As you can see, it is impossible to compose music of any interest without at some point moving the voices. Imagine a similar example in which each of the voices remained on the same note. The result would be completely lacking in both melody and appeal! This movement of voices, however, is far from arbitrary. There are, in fact, very strict and seemingly countless rules that describe the manner in which voice leading must occur. As in geometry, these "axioms" of voice leading were not constructed so that music could be written as a result. The music came first, and the rules merely communicate what has already been proved to work.

The specific rules themselves were derived from the work of certainly the most widely renowned and prolific composer of all time, Johann Sebastian Bach (Kostka, 75). In his capacity as a church organist Bach harmonized 371 pre-existing melodies, setting them within a four-part harmonic progression to be sung by a chorus (Greenburg). At the time he had no textbook to describe the method for doing so, no rules to stick by so as to achieve a pleasing result. He simply wrote what sounded good and, being a genius, the results were unprecedented.

Geometry vs. Voice Leading:

The rules of voice leading that are taught in classrooms today were derived from Bach's chorales. Just as Euclid's axioms do not determine the nature of geometry, these rules were not established so that music could come into existence: they merely describe what sounded good to Bach all those years ago. They can be used, however, to replicate his success and build upon our knowledge of exactly what it is that the human ear finds so pleasing in well-established harmonic structure.

Euclid's five essential axioms are simple enough, and are as follows:

1. For every point P and for every point Q not equal to P there exists a unique line l that passes through P and Q.
2. For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.
3. For every point O and every point A not equal to O there exists a circle with center O and radius OA.
4. All right angles are congruent to each other.
5. For every line l and for every point P that does not lie on l there exists a unique line m through P that is parallel to l. (Greenberg, 14-19)

These axioms describe the system of geometry that was prevalent in society and the world as Euclid was acquainted with it, and they are intended to be self-evident. From these axioms can be deduced propositions which, with the support of a limited number of undefined terms that will be discussed later, compose the framework of what has come to be known as Euclidean geometry.

Interestingly enough, there are also five elemental principles of voice leading. One difference between these postulates and those of Euclid that must be kept in mind is that the

axioms of voice leading were not written by Bach himself, but merely derived from his music by later generations of scholars and theorists. Before they can be explained or examined, however, a small number of defined terms must be contributed to the axiomatic system at hand. Here is a simple and somewhat imprecise list, reference to which will be made throughout this paper:

Rhythm: the placement of notes in relation to the beat.

Stepwise: the movement of a line by a single scale degree in either direction.

Leap: non-stepwise movement.

Numbering: the seven degrees of a scale can be referred to by number (1-2-3-4-5-6-7).

Chord: the simultaneous use of three or more scale degrees.

Harmony: the notes belonging to a chord.

Triad: a chord containing scale degrees separated by a single digit (ex. 1-3-5).

And now, with this foundational information in mind, we may address the axioms themselves:

1. *Rhythm.* Keep the rhythm simple, with most durations being equal to or longer than the duration of the beat. The final note should occur on a strong beat.

2. *Harmony.* Every melody note should belong to the chord that is to harmonize it.

3. *Contour.* The melody should be primarily stepwise. The shape of the melody should be interesting but clear and simple, with a single focal point, the highest note of the melody.

4. *Leaps.*

a. Avoid augmented intervals, 7ths, and intervals larger than a perfect octave (P8). Diminished intervals may be used if the melody changes direction by step immediately after the interval. [Augmented, diminished and perfect are advanced theoretical concepts, unnecessary in the present context.]

b. A melodic interval larger than a P4 is usually best approached and left in the direction opposite the leap.

c. When smaller leaps are used consecutively in the same direction, they should outline a triad.

5. *Tendency Tones.* In tonal music 7 has a strong tendency to move up to 1. An exception to this is the scalewise line descending from 1: 1-7-6-5. The only other tendency tone that needs to be considered is 4, which often moves down to 3, but not with the regularity with which 7 goes to 1. (Kostka, 75-76)

The most apparent difference between these two sets of axioms, apart from the subject matter, is the language. The first set of rules seems very much to be set in stone. Two points DO determine a line, and that is the end of the matter. This is not a truth of the system *most* of the time, or with the exception of certain cases: it is simply a fact of Euclidean geometry. Any model that claims to represent such geometry must be consistent with every one of the axioms, for without consistency there is no model.

The principles of voice leading function in a different fashion. As is apparent from the language used in the stated rules, there is an exception for every statement made. Although it is true that the seventh degree of a scale nearly always leads to the first degree, or tonic, there will always be a plausible exception to this rule that must be incorporated into the system of music theory. In this way, voice leading is a manner of axiomatic system that can contain numerous contradictory models that all, at the same time, represent the system with great fidelity and accuracy.

Judging from the success and failure of my predecessors, it seems that most attempts to axiomatize a system outside of the realm of math are doomed, especially when one is dealing with creative license and the arts. It is simply impossible to set definite and unquestionable rules to a discipline that relies on human intuition and emotion, the regions of our psyche that are still

far from understood (Haberman). It may be asked, then, what the purpose of even attempting to axiomatize such a discipline might be. While and exercise in futility, such an endeavor is not without fruitfulness. By attempting to set an artistic form to a set of descriptive rules, one can learn more about that system and come to understand it at a deeper level as the pieces fall into place and a general form emerges from what appeared previously as seemingly formless expression.

Undefined Terms:

An axiomatic system cannot function without certain undefined terms, for without the universal acceptance of certain principles there is no solid ground on which further knowledge can be established. The undefined terms of Euclidean geometry are point, line, incident, between and congruent. Throughout the development of the system these are the terms that must be relied upon to provide an essential substance to the axioms and propositions.

The corresponding undefined terms of voice leading—at least in the system as it is to be presented here—are beat and scale. Other terms are to be defined in relation to these, and it is hoped that the reader can at least conceptualize each of the preceding. There is yet another undefined term, however, that must be incorporated into the system if a more complete understanding is to be achieved.

Voice leading functions under the authority of one overarching undefined term, and that is good sounding. No matter how carefully you follow the rules, the final judgement just comes down to whether the result sounds good or not, and there is no way to accurately define what this means. Just as humans have a wordless concept of what exactly a point is, we all understand exactly what sounds good in terms of tonal music and what does not without possessing any means of describing that understanding. While taste and the development of appreciation can play into our acceptance and enjoyment of more abstract forms of music, the difference between a harmonic progression constructed according to the axioms presented here and one constructed

with complete disregard for those axioms would be clear to anyone. We must finally rely on our intuition to determine what works and what fails to work, when the axioms hold and when there are exceptions.

The Subtopics of the System:

The development of incidence, betweenness and congruence geometries was the step that had to be taken in order to fill the gaps in Euclid's geometry as outlined in his *Elements*. The five original postulates established an excellent starting place, but they lacked the substance necessary to provide the structure for a truly airtight system. Likewise, the five introductory rules of voice leading are merely general concepts that must be kept in mind if the composer wishes to meet with success. Molded on their influence are innumerable rules of great detail- the propositions of voice leading- that dictate the shaping of a harmonically voiced phrase.

Just as neutral geometry can be divided into three areas of concentration, tonal voice leading can be divided into the study of doubling, tendency and inversion. The chords available in elementary tonal harmony generally contain three separate tones. Because choral harmony consists of four separate parts, one of these tones must occur in at least two parts, the placement of which is referred to as doubling. The two essential axioms of doubling are as follows:

1. All members of the triad are usually present. The final chord is sometimes incomplete, consisting of a third and a tripled root (first scale degree).
2. The root is usually doubled. The leading tone (seventh scale degree) is almost never doubled because it is such a strong tendency tone. (Kostka, 91)

These two rules are strongly associated with and influenced by another foundational aspect of voice leading, that of parallelism. Parallelism is a simple concept, which is why I have not included it as a complete subtopic of the axiomatic system. The statement of parallelism is

this: the movement of voices at an interval of a fifth or an octave from each other in parallel motion is unacceptable. Voices at an interval of a fifth will be separated by three digits (ex. 1-5). Voices at an interval of an octave will be of the same scale degree (ex. 4-4). Parallel motion consists of the movement of those voices by the same number of scale degrees in the same direction. Here is an example of unacceptable and acceptable movement (Kostka, 83):

This over-arching principle influences every area of the discipline without exception, and is perhaps the most seldom broken rule of voice leading. As with any statement that is made within the system, there are exceptions that that must be considered as acceptable, but they are very few. The integration of this principle into the axioms of doubling is readily apparent. It is not a good idea to double the leading tone because such doubling would lead to imminent parallel octaves, the result of both leading tones moving up one half-step to the tonic. These first two rules are just the most basic guidelines regarding doubling. A further proposition, for example, states that while it is preferable to double the root of the chord, it is better to double the fifth than the third.

Unlike the specific areas of neutral geometry, which form a progression from incidence to betweenness to congruence, the areas of voice leading are all bound up in each other. The second doubling axiom itself is based on the rules of tendency, for if the leading tone possessed no drive towards the tonic the rule would lose its foundation and meaning. Several other propositions of doubling are directly influenced by inversion. One states, for example, that when moving between root position and first inversion chords it is necessary to alternate doubling the third and the root or fifth. Another states that it is always best to double the fifth of a chord in second inversion. The principles of inversion, however, will be dealt with later.

We will now move on to the axioms of tendency. Tendency takes on two roles, one applying to individual tones and the other to entire chords. As is stated in the original set of postulates, the seventh scale degree has a tendency to move to the tonic and the fourth scale degree a lesser tendency to move to the third. While this is the essential axiom of tendency, there exist other propositions within the subtopic, such as the tendency of the second scale degree in a V^7 chord (to be later defined) to move to the tonic.

It has now become necessary to add further structure to this system. It has already been mentioned that the degrees of a scale can be named through the process of assigning a numerical value to each. A triad has also been defined. It must now be stated that the triad built on each subsequent scale degree can be named after that scale degree. For example, a triad built on the third scale degree is referred to as a iii chord. Upper and lower case Roman numerals denote major and minor triads, but this aspect of music theory need not be dealt with in the present context. Also, a fourth member can be added to the top of any triad, the result of which distortion is referred to as a seven chord (ex. 5-7-2-4 = V^7).

The tendency of chordal structure can be described in words, but it is better represented by a simple diagram (Kostka, 116):

		[↓IV]	[↓vii°]	
[iii]→	[vi]→	[ii]→	[V]→	I

As one can see, the triads based on certain degrees of a major scale possess an innate tendency to move to certain other triads. This movement is impossible to defend or prove: it simply sounds good. If you were to listen to a progression, for example, in which the iii chord moved to the V, it would sound all wrong, even to the untrained ear.

There are, once again, exceptions to this model of progression. It is quite common, for example, for the V chord to move to vi instead of I, an occasion referred to as a deceptive cadence. It is also acceptable, if less common than the alternative, for iii to move to IV. The propositions of tendency are again both numerous and intertwined with the other two subtopics of

the voice leading system. One proposition, for example, states that any chord that tends towards the V chord can lead first to a second inversion I chord. Another notes that the I^7 will resolve to the IV chord. Yet another states that if vii° (° denotes diminished, an unnecessarily advanced concept) and V are used sequentially, the V will nearly always assume the latter position.

The final subtopic of the voice leading system is that of inversion. It has already been mentioned and incorporated on several previous occasions, but I will now describe it in full. There are three possibilities of inversion when one is speaking of a basic triad: root position, first inversion and second inversion. When a triad is in root position, the root of the chord occurs in the bass part. In first inversion, the root is moved to the upper voice of the chord so that the third is in the bass. In second inversion, the third is in turn moved to the upper voice, leaving the fifth in the bass.

The only axiom of inversion is simple enough: it is to be used to enrich the harmonic structure and add interest to a progression (Kostka, 125). There are, of course, a number of propositions that build upon and add detail to this general rule. One states that inversions can be used in base arpeggiation (the movement of a voice between members if a triad) to avoid constant interval leaps. Another illustrates the use of first inversion chords as substitutes for root position triads for the dual purposes of adding interest to the bass line and lessening the importance of the V and I chords. The propositions of tendency now come into play, for without a sensible movement from one chord to another there can be no cadences and no use for first inversion chords in the role of weakening otherwise powerful cadential members.

Other propositions include the statement that the vii° chord must nearly always be present in first inversion, the comment that the vi chord in a deceptive cadence must be in root position, and a mention of the use of second inversion chords to establish stepwise movement in the base. Inversion is a topic absolutely saturated with propositions, but there is no need to dwell on them further in this context.

Beyond the Basics:

Both Euclidean geometry and tonal voice leading hold in common the fact that they are just the beginning. While Euclidean geometry was almost universally accepted as truth at the time of its conception, later developments led to the hypothetical replacement of Euclid's fifth postulate with the hyperbolic parallel postulate, the foundational axiom of hyperbolic geometry. Hyperbolic geometry still finds its roots in neutral geometry and probably could not have developed without Euclid as a predecessor.

We may now incorporate once again the paper writing analogy introduced at the opening of the discussion. As we first begin to cultivate out writing skills, we are provided with specific rules and form to which we must adhere. Many students, for example, are taught that a basic essay must include five paragraphs, the first of which is to introduce the topic and the last of which is to conclude the paper through a restatement of the thesis (I fear that my paper has somewhat exceeded the limit placed on paragraphs). Other rules speak to the issues of paragraph structure and appropriate grammar. As we mature are writers, however, it can be hoped that we advance beyond the confines of these elementary principles and express ourselves in a more creative and illuminating fashion. Such development, of course, cannot occur without a primary understanding and mastery of the essentials. We must thoroughly understand the rules before we can break them with any hope of a positive and enriching outcome.

Likewise, the composition of music has evolved far beyond the elemental rules of tonal voice leading presented in this paper. Unlike further developments in geometry, however, 20th century music has come to completely disregard nearly every axiom and proposition I have listed (the same can be said of writing, as well as nearly every other creative endeavor). It would seem to the casual observer that there is no relation whatsoever between the music composed in the present day and the harmonic, orderly chorale style of the Baroque era.

While this is true in a purely superficial sense, it must also be noted that every contemporary composer has studied at great length the elements of tonal harmony and is

thoroughly familiar with the rules presented here. It is not until one can understand and manipulate these rules that one can successfully break them. All music composed today is still rooted in the tradition established by Bach nearly 500 years ago.

Conclusion:

Perhaps the most essential element held in common by music and geometry is that of mystery. Both have existed since time began, influencing our lives before we even had a name for them. Geometry is a fundamental component of the universe we live in. It can be noted everywhere from the simple behavior of lines scratched on a rock to the ongoing scientific and mathematical debates over the geometric shape of our universe (Joyce). Likewise, it is difficult to escape from music, an element of our lives that ranges from the simple sound of hands clapping to the song of birds to the complex and dynamic symphonies of Beethoven.

Apart from being merely ever-present, there is also an aspect of music that will never be defined or explained: its indiscriminate appeal to the human race. How is it that simple sounds, when arranged in the proper setting, can have such power over us? Music can make us laugh or cry. It can be relaxing, disturbing, energizing. It plays havoc on our sensations and emotions, recalls forgotten memories and moves us in a way that nothing else can.

I spoke before of the failure with which any attempt to axiomatize a creative discipline must meet. Because the axioms of voice leading are conditional and riddled with exceptions, the system itself is inconsistent and, one might debate, therefore meaningless. At the risk of undermining all of the work I have done and all that has been presented in this paper, I wish to propose that there is only one true axiom of voice leading, and that that one axiom is in fact based on the over-arching undefined term mention much earlier. To state it quite clearly, the ultimate axiom is this: a harmonic progression must sound good to someone, somewhere, at some time. Only a system of voice leading reduced to this single postulate can be completely devoid of contradiction and therefore consistent in an axiomatic state.

Bach discovered what sounded good but he didn't discover why, and it seems unlikely that anybody ever will. How is it that we expect each chord to lead in a specific direction, discovering an unsettling sense of incompletion in one cadence and a firm sense of resolution in another? Why do parallel fifths upset the balance of a progression, and why does the leading tone insist on resolving to the tonic? The tonal scale is itself a mystery rooted in the physics of vibration, but that is a subject for extensive investigation that has no place here.

It cannot be denied that no matter to what extent we take our analysis and definition of music we will never truly know what it is and what role it plays in our universe. The same can also be said of geometry. While it is possible to axiomatize geometry to an impressively comprehensive extent, there is still a fundamental sense of mystery lurking behind the facts. One obvious case of this is the mere existence of Euclidean and hyperbolic geometries. If we knew all that there was to discover about geometry, then we would know for certain whether Euclid's fifth postulate held or failed to hold, and only one of the systems would exist. The mere condition of the universe in which we live, whether Euclidean or hyperbolic, is unknown. Music, like geometry, is a concept that, while enduring of intense study and examination, is ultimately beyond our understanding: a limitless field in which we can but dabble, amusing ourselves with the notion of comprehension.