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Honors Math

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To Choose or Not to Choose?: Proof in Economics

> Cornell has two sets of faculty tennis courts, one outdoor and one indoor. Both courts have a seasonal charge to them, but the indoor court has an additional fee of $15 per hour. This extra cost reflects the addition costs of heat, electricity, and building maintenance. The indoor facility opens in early October, a time when the weather can be anything from sun and mild temperatures to snow. The outdoor courts remain open, weather permitting, until early November. Say you are committed to pay for an indoor court at noon on Sunday, October 25. It is a warm and sunny day. Which court should you use, indoors or out (Frank 269)?

Surprisingly this was an actual study taken out of a book on microeconomics called “Microeconomics and Behavior” by Robert H. Frank. When most people think of economics they believe that it is a lot of number crunching, bazaar predictions, and all in all an inaccurate science. The example above shows that economics can be applied in many different fashions and even help us make decisions in our everyday lives. In the problem presented, it was discovered that the majority of people would choose the indoor courts over the outdoor and, yes, even I myself chose the indoor court, but why? In my mind I thought it would be a waste of money to pay to use an indoor court and then use a court outdoors, but where are the losses really coming from? In economics the $15 paid is referred to as a sunk-cost in that it is an amount paid that is impossible to get back. Contemporary economics tells us that we should not factor sunk-costs into our decisions, because there is no way to get back that money. It doesn’t matter which court you use, because those $15 are already gone and there’s no way of getting it back. You paid, in essence for the time, so now it would be in your best interest to use that time in a way that makes you most happy. In a perfect world everyone would choose the outdoor court because they would prefer to be outside rather than in on a nice day.

What is it that Economists Do?

Economics is often referred to as a pseudo-science. Economists attempt to explain human choice in a world with scarcity of time, energy, and material through axiomatic systems, numbers, and models. There is always a limited amount of the things we need or want. At the same time there is a problem with explaining people in a systematic way. While all of these methods of explaining human behavior have clear and provable answers, people are much more variable in the ways they make choices. It is the job of the economist to find the rules that govern the behavior of "rational" individuals (the term rational will be explained in more detail further on). They must then build mathematical and provable axiomatic systems that would react the same way an individual would based on these rules. For the sake of economizing the best use of paper and time this paper will attempt to explain how economists prove ordinal binary preference/relationships. This is a proof assuming when a person is given a choice between two goods they can explicitly state which they would prefer more. Basically, we will see how economists prove what a consumer will choose when given two choices.

Axiomatic Proof

Axioms in geometry are the rules for which every other rule is derived from. Without these agreed upon axioms, a mathematician would not be able to show the existence of a line or how to form a perpendicular angle. Some branches of economics, such as microeconomics, which we will be using, are much the same. They use axioms on human choice behavior in order to show what and how an individual will make a choice between two options, A and B. There are six axioms of rationality pertaining to ordinal binary preference/relationships. This set of axioms also includes some of the axioms used in choices A, B, and C when risk is involved. Risk is the chance of loss as measured by a percentage. While these are important we will focus mostly on the axioms involving a comparison of A and B. The following are the six axioms with brief explanations of each:

Axiom I: Comparability (completeness) – For any pair of alternatives A and B an individual must be able to say that they either prefer A over B, B over A, or they are indifferent between the two choices. For this to be true, the outcomes A and B must be finite/bounded and within the realms of comparison (trichotomy).

Axiom II: Transitivity (consistency) – For any set of three alternatives A, B, and C, if A is preferred to B and B is preferred to C, then A is preferred to C. For example, this rule would be violated if someone were to prefer their math class to their biology class and prefer their biology class to their English class and yet prefer their English class over their math class.

Axiom III: Independence of Irrelevant Alternatives – This axiom states that the binary relationship that holds between any two events A or B is not affected when each is combined in a gamble with an arbitrary third event, C. A gamble is a situation where the outcome is uncertain. Thus if:

A = B and probability is $\alpha 1$. $\alpha 1$ is a percentage measure of the likelihood of an event occurring. Now set up two gambles G1 (A, C;$\alpha 1$) and G2 (B,C;$\alpha 1$)

Then, G1 (A,C;$\alpha 1$) = G2 (B,C;$\alpha 1$)

Axiom IV: Continuity (measurability) – If B is preferred less than A, but more than C (A ϕ B ϕ C) then there is a unique probability α such that the individual is indifferent between B and the gamble where A occurs with probability α and C with (1-α). Symbolically, let A ϕ B ϕ C, $\exists$ Probability = α B = G(A,C;α). This means that you can measure the adjustment need to change a consumer's preference for an event by changing the likelihood that it will occur.

Axiom V: Ranking (dominance) - If an individual's preferences for outcomes B and U lie somewhere between A and C, then we can establish a set of gambles such that the individual is:

i. Indifferent between B and a gamble involving A and C occurring with some probabilities $\alpha1$ and (1-α) respectively.

ii. Indifferent between U and a gamble involving A and C occurring with probability $\alpha2$ and (1- $\alpha2$) respectively. Furthermore,

If $\alpha1 > \alpha2$, then B ϕ U. Or if $\alpha1 = \alpha2$, then B = U.

This is a continuation of axiom V, but this describes the ability of a consumer to say that they would prefer one gamble over another. It is in the same spirit as axiom II.

Axiom VI: More is preferred to less. This is the idea that everybody would rather have more of something than less (Marcose 2-3).

With these six axioms combined two men by the name of Oskar Morgenstern and John Von Neumann established a fundamental theorem of economics that states: "under uncertainty rational individuals maximize expected utility of wealth" (Marcose 3). Utility is a rating of happiness and rationality is defined as the first two axioms. While it is possible to go on to show how these two individuals arrived at this theorem doing so would require a list of definitions of economic terms and functions. It also relies on an intermediate theorem that establishes the correspondence between ordinal preferences (the ability to rank preferences) and a cardinal utility function (the ability to apply value to rank) (Frank 95). Like many proofs in geometry, the Morgenstern-Neumann theorem was done using an RAA hypothesis where it was assumed that an individual would not attempt to maximize their expected utility. In the real world this theorem would make sense to most individuals. The theorem states that when confronted with options the consumer will attempt to pick the option that we would expect to give them the highest utility (greatest pleasure). Using these axioms and theorems, economists use other means, namely graphs, to explain what an individual would choose given two options A and B.

Proof Using Graphs

Graphs are a clear and organized way of presenting information. They can be seen in use throughout the academic arena from lab reports and psychology reports to statistics classes. In economics they are used in the same manner and due to their form they are dynamic and adaptable. In our graph we will show how a consumer chooses between product A and product B. This will be done in a Euclidean plane. The graph is built as a model, but the assumptions taken into account for the graph will be the axioms previously stated. Without them it would be impossible to draw the proper assumptions from the graph.

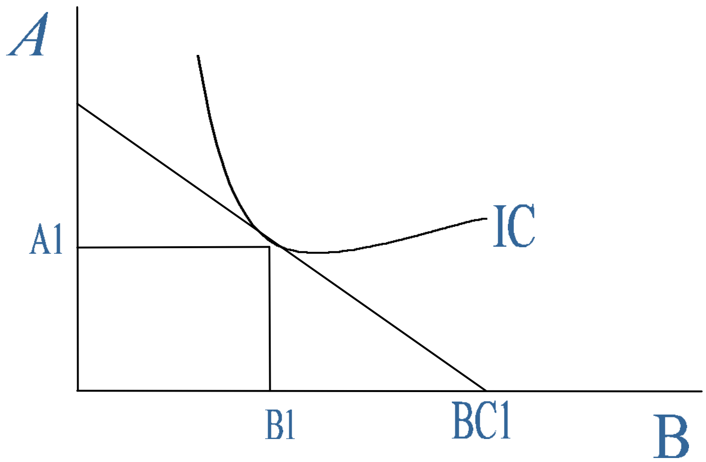

Figure 1. Traditional budget constraint and indifference curve graph.

The first task is to set the axes. The x-axis will reflect the amount of B you are able to purchase while the y-axis will reflect the amount of A. The straight line from one axis to the other is referred to as the budget constraint line. Any point underneath this line is referred as the opportunity set or the set of all possible combinations of A and B that you can buy. This line is different for each consumer, because it is based on three known facts: the price of A, the price of B, and the income of the consumer. To find where this line will intersect the x-axis you simply divide the income of the individual by the price of B and you do the same for the y-axis, but divide the income by the price of A. The only curved line that appears on the graph is referred to as the indifference curve (IC). It is referred to by this name is because a person would be equally happy to have any combination of A and B on that line. The IC is placed based on a consumer's personal preference. The further up and to the right the IC gets the happier an individual is because they are allowed more of both goods and based on our axiomatic system more is better. If a person only likes A, then the IC curve will be incident only where the y-axis and budget constraint line are incident showing a total preference for A. For the IC curve shown, the consumer will select A1 amount of product A and B1 amount of B. What economists then attempt is find out what this consumer would do if their income increased, decreased, or if the price of A or B were to fall or rise. For our example we will consider the case that this consumer's income increases.

An increase in income would shift the budget constraint curve to the right from BC1 to BC2 (see figure 2), because the consumer would be capable of affording more of A and B. This also means that this consumer will be capable of reaching a higher indifference curve in the form of IC2, so this will also move up and to the right. This consumer will purchase A2 amount of A and B2 amount of B. We have proven what amounts of A and B this consumer will purchase if their income rises, but does this really work? One important aspect that was not mentioned was the use of the rule of ceteris paribus, which means that all else was held equal. We arrived at the answer we did because we only changed income. In actuality we could have changed many variables including the prices of A and B and this individual's tastes and preferences (IC), but if we did it would still be possible to arrive at some amount of A and B chosen by this individual. The benefit of this is that it can be repeated for any individual with any income and any tastes and preferences as long as we use the original axioms.

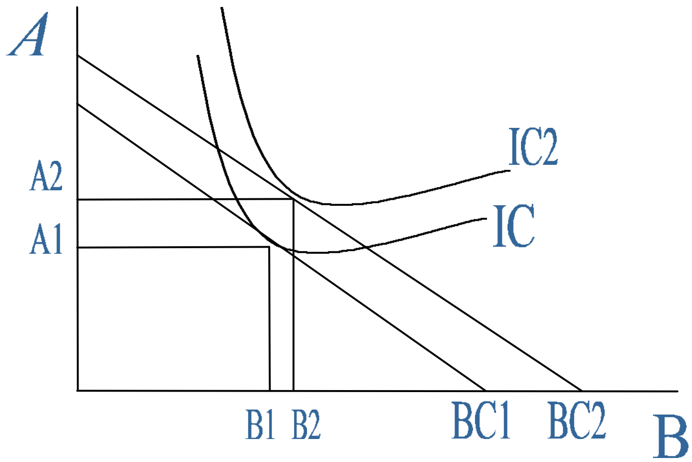

Figure 2. Traditional budget constraint and indifference curve graph with increase in income.

One problem that many economists face is that no individual consumer actually constructs these curves before making a decision. For this problem there are two important facts to note. First, economists understand that people don't actually create these graphs in order to decide how to spend their wealth. The graphs are made to simulate a person's response mathematically. The point is to take the same inputs for a person and a graph of that person's income and preferences and to arrive at the same amount of A and B at the end. We prove that the graph is a model of human behavior. It simulates, in a structured way, how an individual chooses. Secondly, this model is based on what an individual perceives to be the best for them. This process is conducted before the fact. Economists can't predict what will bring the greatest utility, but they can predict what we would perceive to bring the greatest utility. But how does an economist go about proving that these graphical assertions are valid and mirror the real world?

Econometrics

Econometrics is the statistics of economics. While graphs work very well at showing results when changes are made econometrics allows economists to show that these graphs are valid interpretations of human behavior. For our example this is difficult to show because there is no known situation where a person spends all of his/her income on two goods. However, we can change the graph slightly to work for real life situations.

Let's assume that instead of having a choice between two goods, A and B, you have a choice between product A and OG, which stands for all other goods. The same graph and logic would apply, but instead of a comparison of two products, A and B, it is a comparison of one product, A, to all other possible products you could purchase, OG (other goods). One good, B, is replaced by all of the other goods you could possibly buy other than A. The validity of this type of graph is tested by first figuring out how much income the person makes, the price of good A, and the price of all other goods (which could be represented as the individual's entire income), and how much the individual prefers A to all other products. The amount spent on good shows the amount of preference for good A. Then you review statistics over a certain time period of how much of product B the person consumed. It is best to do this over a very short period of time or a very long period of time so as to avoid a change in tastes which can average out over the long run. This is not an axiom, but a condition that allows the results to be clearer. You then compare the numbers to the graph and see how close the graph is to being correct. This is often done using whole populations of people or different governments to predict how much they are likely to spend on a given good (e.g. military). This also works much better on a large scale, because of the law of large numbers, which creates figures that are less likely to change. The law of large numbers says the larger sample you take the closer the result will be to the true average. Econometrics allows economists to take their theories and graphs and apply them to real like statistics to see if their theories hold. It is this connection between simple theory and the complex interplay of real life that poses the greatest obstacle for economists.

Conclusion

Economists draw from many areas of proof in order to show what they believe to be true. From RAA hypothesis to the use of statistical evidence, economic theories are turned into real life applications and real life is turned into an existence that is much more predictable and measurable. Although often referred to as a pseudo-science, economics provides a basic set of rules that may be checked, models that may be replicated, and the ability to reproduce the same results, either theoretical or statistical, just as any other type of science. The difficulty is found in the fact that we are all different, but the solution to the problem is seen through our one similarity, the mutual pursuit of happiness.

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