Kant and the "Argument From Geometry"

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Math 300

I. Introduction

Immanuel Kant in his *Critique of Pure Reason* (1781, 2nd ed. 1787) aims to effect a sweeping change in how human knowledge is understood with his "Critical" philosophy. Kant has often been called the greatest of modern philosophers; his body of work included treatments of epistemology and metaphysics (how it is that we can have knowledge and what "reality" consists of, respectively) as well as morality and aesthetics, the study of artistic taste. Kantians and neo-Kantians abound even today—Kant's influence on the trajectory of modern philosophy is difficult to exaggerate. Here, though, I focus only on one aspect from the *Critique of Pure Reason*, Kant's major epistemological work: the so-called "argument from geometry." This argument lies in the "Transcendental Aesthetic," a portion of the *Critique* in which Kant investigates the nature of what we can know *a priori*—literally, "prior to" any experience of the world. As Kant is perhaps among the most difficult (and most German, in his coinages) of philosophers, I will try to be as gentle as possible on the reader. It is some tough going, however—Kant is many things, but he is very rarely easy.

II. The Transcendental Aesthetic

a) Concepts and intuitions

Kant divides all of cognition—that is, intellectual activity—into two categories, *intuitions* and *concepts*. *Intuitions* are the objects which we find "given" to us in

sensation from the world—they originate *outside* the mind and its cognitions. They are necessarily particular and individual—an intuition is a *singular* object. As such, reasoning from intuition is limited by its particular nature, in that the most reasoning from intuitions may aspire to is mere inductive certainty—the probability that a given proposition is true, given the number of particulars observed. Intuitions are processed by the faculty of the sensibility and can be understood best perhaps by identifying them with the perceptions and sensations that the mind receives from outside it. In contrast, *concepts* are employed by the faculty of the understanding and are products of the mind. Concepts are used by the mind to "think" objects—they are the constructs of the mind imposed upon the world presented to it to create some sort of perceptual order. Concepts are always general and universal and thus only reasoning from concepts can yield what Kant calls "apodictic certainty"—that is, universal validity of the sort we find in geometric proofs. (CPR, A20/B34)

The distinction between concepts and intuitions is a crucial one for a grasp of Kant, so it is worth considering a few examples. Mark Risjord asks us to consider the act of seeing a dog. Now there is a part of this viewing of the dog which is certainly outside the mind—we are presented with a determined image of a dog, with a definite shape, color, etc. This is the *intuition* of the dog seen—it is particular, individual, and given to the mind. The *concept* of dog is that which the mind uses to categorize and order what it is given—the dog-concept has no particular color, shape, etc. because it must share in the qualities of all dogs. Further, the dog-concept gives us information that the mere intuition could not—that dogs bark, that they (usually) have four legs (even if some are obscured from view), that they are mammals—these are qualities common to all dogs,

and result from our projecting the dog-concept in the appropriate circumstances (i.e. when we believe ourselves to be in the presence of a dog). It might be useful to note here that what we are presented ("given") in sensation is generally not at all sufficient to explain our knowledge and action in the world—all of intuition is presented in more-or-less two dimensional form, with no given way to necessarily process it. How, for example, are we to know that objects diminish with distance? If we were to rely only on intuition, we could never know such a thing—it is only once concepts are employed to differentiate ("condition" is Kant's word, often) intuition does what we see before us begin to make sense. Kant concludes that we must *ourselves* put constraints onto the world we experience, that knowledge is not a mere passive activity as in just receiving what the world gives us, but rather that it is an *active* endeavor—that we actively shape our experience by projecting concepts into the world, which we then see as *part* of that world. Kant's transcendental idealism takes this insight, that we actively shape the world we experience, a step further, as we will see below.

Particularly given the distinctions between intuitions as particular and concepts as universal, it may be useful to note that concepts can be thought of as sets or classes, and intuitions as the individual members of those sets.

There is a bit more terminology we have to master before we can examine Kant's Transcendental Aesthetic.

b) *a priori* and *a posteriori* knowledge

As briefly discussed earlier, knowledge for Kant can be divided another way: into *a priori* and *a posteriori* knowledge. *A priori* knowledge is knowledge that we have prior to any experience of the world—it is knowledge independent of any particular experience

or other. *A posteriori* knowledge, in contrast, is knowledge *after* experience, derived from it. Examples of purportedly *a priori* statements—that do not appeal to any experience for their truth—are things like "the parts of the parts of a thing are parts of that thing," and various mathematical and logical judgments. It has been argued that '5 + 7 = 12' is true independent of any particular experience, though this is contentious.

c) Analytic and synthetic judgements

Judgements for Kant come in two kinds: analytic and synthetic. A judgement is made of two parts: a subject and a predicate. A subject is what the sentence is about; the predicate is what is said *about* that subject. (So in the sentence "Jane likes Bobby," the subject is Jane and the predicate is "likes Bobby." Similarly, in "The dog jumped over the fence," "the dog" is the subject and "jumped over the fence" is the predicate. You get the idea.) An analytic judgement is one whose predicate is contained within the subject—a typical example given is "A bachelor is an unmarried man." The sentence asserts nothing that is not contained in the idea of a "bachelor"—if we know what bachelor means, the sentence provides no additional information. Synthetic judgements, on the other hand, are those which *do* present information not already present in the subject—like, for example, "A bachelor committed the Green River murders," which we would never glean from analyzing "bachelor," no matter how long we persisted.

Relating this to our earlier discussion about intuition and concepts, Kant asserts that knowledge we get from examining concepts is *always* analytic. For concepts are exhausted by the things they contain; without their members, concepts are nothing. So the only things our "concept dog" could yield are analytic statements about what is contained within that concept—e.g. "dogs are mammals," "dogs are furry," etc.

Synthetic judgements require "some other thing" to connect the subject and predicate. This "other thing" is *possible experience*—the possibility of an intuition which relates the subject to the predicate.

d) Pure and empirical intuitions

Intuitions come in two kinds: pure and empirical. An empirical intuition is one which has some sensible content; a pure intuition is one which has no sensible content. Using our seeing a dog example, then, the empirical part would be everything which is sensed—that is, the particular colors, shapes, etc. The pure part of the intuition—and this is a rather difficult thing to grasp—is that which does not correspond to any sensation or other but yet is present in the intuition. Take an intuition and abstract away all particular sensation and you are left with the pure part of that intuition, Kant argues:

> …if I take away from the representation of a body that which the understanding thinks in regard to it, substance, force, divisibility, etc. and likewise what belongs to sensation, impenetrability, hardness, colour, etc., something still remains over from this empirical intuition, namely, extension and figure. These belong to pure intuition, which, even without any actual object of the senses or of sensation, exists in the mind *a priori* as a mere form of sensibility. (A21/B35)

The *matter* of a representation of some object is what Kant calls the sensible component of the representation; the *form* of the representation is that which gives that sensible component order. Kant argues that while the *matter* of all representations is given to us *a posteriori* (that is, through experience or acquaintance with the external world), there must be some *a priori* forms of representation that the mind is able to use to order the matter presented to it.

Hopefully this discussion has clarified Kant's claim that "there are two pure forms of sensible intuition, serving as principles of *a priori* knowledge, namely, space and time." (A22/B37)

e) The possibility of synthetic *a priori* knowledge

We find Kant wondering about the possibility of "synthetic *a priori* knowledge"—that is, knowledge which we can know prior to any experience which yet give us information about the way the world is beyond what is contained in our concepts.

Why does Kant think such knowledge exists? The success and existence of mathematics (and in particular, geometry) convinces Kant that such knowledge is not only possible but is central in mathematical reasoning. Kant argues that pure mathematics are both *a priori* and synthetic: *a priori* because mathematical judgements have a certainty which could never come from *a posteriori* reasoning; synthetic because they give us information about the world not contained in our concepts already.

And how is such knowledge possible? The short answer for Kant is that it is possible because we find that we have projected into the world the pure form of our sensible intuitions—because it is the form which all of our intuitions share, it is *necessary* and therefore *a priori*. However, because these pure mathematical sciences give us knowledge about the world (that all of our perceptions will be in space and time, for example) beyond our concepts, they are also *synthetic*—so we have synthetic *a priori* knowledge. Note that this certainty does not come without cost—because we project the sensible form of intuition, we gain a certain amount of necessity and certainty, but what we cognize is no longer the world as it exists independently of humans. This is Kant's transcendental idealism, which I address next.

f) Kant's transcendental idealism

This is the centerpiece of Kant's metaphysics, and is quite difficult. I will try to do it justice here in the brief room I have.

The short version is that space and time are the mere *forms* of sensible intuition, and do not exist in the world independent of human cognition. They are merely the "packaging" which all of our perceptions of the world comes in. Kant's proof for this rests on several mini-proofs in the Transcendental Aesthetic that do not concern us here. What exists in the world independent of human cognition is what Kant calls the "thing-in-itself," which we can have no knowledge of. Because all of our cognition is necessarily infused with the form of pure intuition—that is, no experience we have of the world is as it really is—all we have are *appearances*, the world *as it appears to us*. By definition, it would seem, we cannot experience the world as it does not appear to us. This may seem jarring, but Kant is able to maintain a fair amount of empirical reality with this ideality of space and time—Kant is able to accept all of physical science while denying that it in fact applies to reality as it exists outside of human experience.

We are now ready to examine the "argument from geometry."

III. The "argument from geometry"

The "argument from geometry" as it occurs in the B (2^{nd}) edition of the *Critique* is as follows:

> Geometry is a science which determines the properties of space synthetically, and yet *a priori*. What, then, must be our representation of space, in order that such knowledge of it be possible? It must in its origin be intuition; for from a mere concept no propositions can be obtained which go beyond the concept—as happens in geometry (Introduction, V). Further, this intuition must be *a priori*, that is, it must be found in us prior to any perception of an

> object and must therefore be pure, not empirical, intuition. For geometrical propositions are one and all apodeictic, that is, are bound up with the consciousness of their necessity; for instance, that space has only three dimensions. Such propositions cannot be empirical or, in other words, judgments of experience, nor can they be derived from any such judgments (Introduction, II). (B41)

This passage, in concert with a few others in the Aesthetic, have caused some commentators—typically those already unsympathetic to Kantian ambitions—to announce that Kant's project was destroyed by the development of non-Euclidean geometries.

See Hans Reichenbach declaring in 1936 that,

> …by the discovery of non-euclidean geometries, by the logical theory of mathematics, by the rejection of the mechanical basis of physics, and by the relativistic critiques of the concepts of time and space… [t]he evolution of science in the last century may be regarded as a continuous process of disintegration of the Kantian synthetic *a priori*. (Reichenbach, 145)

In a similar spirit, English philosopher and preeminent intellectual Bertrand Russell claimed in 1945:

> The transcendental (or epistemological) argument [for the ideality of space and time]… is… definitely refutable. "Geometry," as we now know, is a name covering two different studies. On the one hand, there is pure geometry, which deduces consequences from axioms, without inquiring whether the axioms are "true"; this contains nothing that does not follow from logic, and is not "synthetic," and has no need of figures such as are used in geometrical text-books. On the other hand, there is geometry as a branch of physics, as it appears, for example, in the general theory of relativity; this is an empirical science, in which the axioms are inferred from measurements, and are found to differ from Euclid's. Thus of the two kinds of geometry one is *a priori* but not synthetic, while the other is synthetic but not *a priori*. This disposes of the transcendental argument. (Russell, 716)

These two examples are representative of dismissals of Kantian epistemology and metaphysics due to Kant's reliance on Euclidean geometry and Aristotelian logic (which also figures into Kant's arguments and suffered a similar fate as Euclidean geometry when it was supplanted and augmented by the propositional logic of the 20th century).

Here I will survey potential responses to such arguments and comment briefly on their merits.

IV. Arguments on Kant's behalf

Lisa Shabel, in a compelling article, has argued that the "argument from geometry" has been misinterpreted by commentators. Rather than being an argument that argues from the premise of Euclidean geometry to the nature of space as merely the pure form of our sensible intuition and nothing to be found in the world apart from in human cognition, Shabel argues that the "argument from geometry" takes the ideality of space to have been proven already. Thus, rather than proving the ideality of space by appealing to Euclidean geometry, Kant takes his previous arguments to have shown the ideality of space and deduces from that how Euclidean geometry is possible. This is a significant change in interpretation—rather than Euclidean geometry bearing the greatest burden of the proof of the ideality of space, Euclidean geometry is merely explained using the conclusions of the previous arguments (which we did not survey here). As such, the demotion of Euclidean geometry from the necessary representation of space to merely one geometry among many possibilities need not be devastating for Kant's arguments here—indeed, it may be largely irrelevant.

The thrust of Reichenbach and Russell's criticisms, though, do not seem to hinge necessarily on interpreting the argument from geometry as arguing from the necessity of

Euclidean geometry as a premise to the conclusion of space as the pure form of our sensibility—that is, the ideality of space. Rather, their criticisms seem directed to Kant's conviction that there *does* exist some synthetic *a priori* knowledge in the world, and that we must somehow account for how that knowledge is possible. This is Kant's "synthetic" argumentative structure—starting from what we observe to be true in the world, what must be the case in order to make that so? Kant gives a number of examples of purported synthetic *a priori* knowledge in the Introduction to the Critique—I take Reichenbach and Russell's point here to be that if these examples can be shown to fail, then there is nothing that needs to be explained. The motivation for the entire Critical project, then, is shown to be empty.

Sebastian Gardner in his guide to the *Critique* has another approach, replying to Russell's criticism that geometry as a synthetic *a priori* science has not been realized. Transcendental idealism, Gardner claims, does not imply that space is necessarily Euclidean—so long as we distinguish the "transcendental concept of space" from the space which comprises outer empirical reality—that is, the space we experience. The "transcendental concept of space" is *a priori* and unconditioned like the space Kant discusses. Gardner argues that the shape of the space we experience is *a posteriori*, contingent on the space we happen to experience. (Gardner, 103) The "space" mentioned throughout the Transcendental Aesthetic, then, is this *transcendental* concept of space—this concept of space is presupposed for experience of any other space. This interpretation, however, circumvents Kant's strategy for the Critical project, insofar as Kant's transcendental idealism purports to explain how it is that we can have an *a priori*

science of space which somehow—as if by magic—exactly matches the space we meet with in experience.

It is clear that Kant's project will not be unaffected by the development of non-Euclidean geometries. However, there are a number of avenues of argument open to the Kantian, beyond those presented above. The Kantian can choose to place more weight on non-geometric examples of "synthetic *a priori*" knowledge that Kant gives—famously, that every event has a cause is a good example. The Kantian also may argue that the *existence* of non-Euclidean geometries need not concern us—Kant's argument is about what we as humans can *experience*. It seems it could be plausibly argued that we cannot experience non-Euclidean space, at least in the same way we experience Euclidean space, and so Kant's argument can be salvaged.

It may be challenged here that Kant goes quite beyond stating merely that we cannot *experience* non-Euclidean space—that he in fact states that we cannot *conceive* of a non-Euclidean space. This, evidently, is false—non-Euclidean geometries certainly have been conceived and shown to be consistent.

V. Conclusion

The "argument from geometry" has supposedly borne the burden of proof in the Transcendental Aesthetic for the ideality of space—Lisa Shabel has argued convincingly that this is an interpretive error. As such, I hold that the development of non-Euclidean geometries need not be the fatal blow to Kantian metaphysics that many have taken it to be—there are a number of possible ways to reconcile Kant's project with the existence of alternative geometries.

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