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Basketball, Psychology and Mathematics: Models of Varied Disciplines

Throughout my four years of college, I have been a member of many clubs, organizations, teams and classes, some for days, some for years. But, when I look back on my years at UPS there are three distinct areas of my collegiate career that stick out in my mind: basketball, mathematics and psychology. One might think these are disparate, unrelated aspects of my varied college experience, but they are more related than one might think. Each of these disciplines involves an axiomatic system that does not lend itself to a rigid set of definitions. Instead, a modeling process can be used to understand each discipline and their quest for truth.

One of the most common strategies to discovering truth in mathematics is through the use of an axiomatic system. An axiomatic system is a logical system that possesses an explicitly stated set of axioms from which theorems can be derived (Weisstein, 2004). Therefore, axiomatic systems consist of a collection of undefined terms and concepts that, although we understand the system to be true, contain terms and concepts that are impossible to prove. For example, the undefined terms of neutral geometry are point, line, incidence, betweenness, congruence and continuity (Greenburg, 2003). These are all familiar terms yet we cannot define these words without putting constraints on their meanings, because two different models with the same axiomatic system may have different interpretations of point, line, incidence, etc.

Similarly, the sport of basketball can be characterized as an axiomatic system since there exists an explicitly stated set of rules (or axioms) governing the game in which each game represents a model of those rules. The terms included in the axiomatic system of basketball may include such basics as point guard, forward or center. Although a player may know what these words mean (and even someone who doesn't play basketball might understand what these words mean), they may be interpreted differently depending on the particular player or team, or a particular situation within a particular game. For example, although a player may be listed as a forward on the official program, she may not always play under the basket and at any particular time may perform many of the duties of a point guard (i.e. dribbling the ball up the court, calling out plays, etc).

Each position on a team, each player on a team and even every play during a given game may be similar across different teams but as a whole they represent a model of basketball that will likely be implemented quite differently between each team. Although every basketball player must have a common knowledge of the rules and expectations of a basketball game, they may contain a varied repertoire of basketball knowledge resulting from varied years of experience, leading to different interpretations of the rules at any particular moment. Therefore, we cannot precisely define the game of "basketball" because doing so would put constraints on how one player or team could model their game after the axiomatic system that is basketball.

Whereas geometry uses an axiomatic system to understand the nature of space and basketball uses an axiomatic system to understand the rules and strategy of basketball, psychology uses an axiomatic system to understand relationships between variables. It is a commonly held belief that it is difficult to establish whether a given psychological theory is logically consistent. Therefore, psychology utilizes an explicit, shared conceptual system in order to develop its theories. A cognitive map is one example of a psychological axiomatic system.

Cognitive maps are representations of the elements of an environment and their spatial interrelations, which are constructed within our brain's capacity. These maps are believed to influence behaviors such as the manner in which we navigate through our environment, or our decision-making processes in every-day life. For example, learning and remembering a locale involves analyzing a series of perceptual and sensory impressions, each of which involves data from only a few objects. By organizing these impressions through a process of associating shared similarities common to them all, one can synthesize them into a coherent component. This component can be renewed in a new environment using the most recent information. This process is not merely a matter of memorizing one's surroundings, but rather one which involves spatial reasoning (Yoshino, 1991). Describing the nature of cognitive maps is essential to understanding how people represent, reason and function in their spatial environment and from these cognitive maps (or axiomatic systems) one can form models that include different perceptions or interpretations of a given environment. (Friedman & Kohler, 2003).

Psychologic is an axiomatic system, using cognitive maps, in which conceptually necessary propositions are embedded in psychological theories and hypotheses, as well as in ordinary language (Smedslund, 2002). Psychologic intends to formulate what is true in psychology. The vocabulary of ordinary language includes numerous, vaguely related terms. By selecting a limited number of terms, restricting them to their core meanings and distinguishing between undefined and higher-order terms, one is left with an axiomatic system. Therefore, since the axiomatic system contains undefined terms, axioms, and formal proven propositions (corollaries and theorems), psychology then has a conceptual framework that is constrained by the semantics of language.

Inherent variability in the domain of psychology makes it impossible to establish necessary truths, which is similar in Euclidean geometry. This can be better understood through an example: *In Euclidean Geometry, "The sum of the angles in a triangle is equal to 180 degrees."* If the sum of the three inner angles was measured and equaled 181.90 degrees, what would this tell us? Either it would indicate that a theorem in Euclidean geometry would need to be revised or that one of the hypotheses used in deriving this result was faulty. When the results deviate from the prediction, either the primary hypothesis or one of the auxiliary hypotheses must be incorrect (Smedslund, 2002). A similar example in psychology, which is the basis for my Senior Thesis, may help to understand this better.

Researchers have hypothesized and proven that children who are involved in after-school activities (sports, music, dance, etc) have higher academic achievement than those children who go home after-school and do not participate in organized activities (Jordan & Nettles, 2000). Say I was to hypothesize the following: *Children who care for themselves after school will have higher academic achievement than those who participate in after-school activities.* My results indicate that 99% of children who go home after-school with no organized activities perform *worse* on tests of achievement than those students who are in a structured, after-school homework program. Since my results deviate from my prediction, either my primary hypothesis is at fault (which is probably the case!) or one of the auxiliary hypotheses is incorrect.

Psychologists typically formulate their hypotheses without concern for the conceptual relatedness of the variables involved. Conceptual relatedness can be defined as "Something follows from the truth value of P about the truth value of Q and/or something follows from the truth value of Q about the truth value of P" (Smedslund). A non-psychological example in the sport of basketball characterizes this well; it does not follow with certainty that for a team to be successful, all the individual players on the team must be the "best" players. Similarly, it does not follow with certainty that having all the "best" players on a team will result in a team being successful. But, given no other information, the likelihood that a team is successful is increased if one knows that the team is comprised of all the "best" players and vice versa. The inferences made are determined based on the meaning of terms but not on the interpretation and results of the combined terms, or on the truth value within the given terms.

In the previous example, the inference is made that the team with the "best" players will be successful because one would interpret "best" player as meaning someone who could dribble, score, rebound and defend. Assuming a team is comprised of multiple "best" players, one would generally infer that ultimately this team would be successful based solely on knowing this team is comprised of "best" players. But, if someone were to watch this team of "best" players in a game, they might see that the players together were not successful and the statement *all the "best" players on a team will result in a team being successful* would therefore be false.

The sport of basketball is an ever-changing model in which every game, every play and every player is different from the one before and the one after. Similarly, psychology is an ever-changing model as well, where no two research subjects, environments or studies are the same. Together, basketball and psychology are like mathematical models of axiomatic systems. While the mathematician works to discover truth behind his/her systems, a basketball player and psychologist are always changing to the interpretations and implementation of their respective models.

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