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The Truth about the Liar

> *"One of the most remarkable features of human language is our ability to use and understand expressions never before uttered."*
>
> ~*Barwise and Perry*, Situations and Attitudes

Math is said to be the universal language. Words strung together in a certain way, create sentences, which, in turn, contain information. However, there is something hidden in this method of communication. The success of communication depends on whether or not the two subjects communicating share a common knowledge. In other words, they each know the situation in which the sentence is taking place, and to what the information refers. The Greek philosopher, Epimenides, generated one of the most puzzling paradoxes that have stumped mathematicians, logicians, and linguists, for almost 2000 years – The Liar Paradox. (Devlin, pp 256) However, a new way of thinking, called Situation Theory, was developed in 1981 by two professors of the Center for the Study of Language and Information at Stanford University, Jon Barwise and John Perry. (Kovacs, pp 44) It was not until then, that we have seen a plausible solution to this paradox.

Situation Theory was created during an effort to "provide a richer, more honest semantic theory of natural languages." (Seligman and Moss, pp 241) It is the mathematical theory of meaning and information. Mathematician, Keith Devlin, also a professor at Stanford University, provided the first attempt to formalize situation theory in 1991. Devlin brings up the important issue that while we are surrounded by information, none of us can easily give a definition of what information actually is. He came up with the term "infon" to describe discrete chunks of information. An infon can be compared to elementary particles, which form to create matter. An infon is denoted by:

$$\sigma = \langle\langle R, a_1, a_2, \ldots, a_n; p \rangle\rangle,$$

where σ is an infon, R is the n-place relation, $a_1, a_2, \ldots, a_n$, are objects appropriate for the respective arguments of the relation R, and p is the polarity. The polarity either takes on the value 0 or 1: 1 if the objects $a_1, a_2, \ldots, a_n$, do stand in relation R, and 0 if they do not. However, an infon itself cannot be true or false – its truth value depends on the situation it is found in.

Shortly after Barwise and Perry published this new theory, Barwise and fellow professor, John Etchemendy, published an essay called *The Liar: An Essay on Truth and Circularity*, applying situation theory to different variations of the Liar paradox. Barwise and Etchemendy provide models for two different definitions of a "true proposition" – one presented by Bertrand Russell, called Russellian Propositions, and another by J.L. Austin, called Austinian Propositions. Russell believes that a proposition is true when there are facts that make it true, and is false when there are no such facts. (Barwise and Etchemendy, pp 75) Because Russell's conception is limited in some cases, we will focus on Austin's method, which has been found to have more productive applications to solving linguistic paradoxes. Austin suggests there is always a contextual parameter for every sentence or statement.

So where does situation theory come into play? It turns out that Barns and Barwise discovered that Austin was on the right path, realizing that the context and situation of every proposition plays a very important role in how it is interpreted. Every proposition (which contains information, and in turn, consists of an infon) has a context or situation, s. In notation following the theory, $s \models \sigma$, reads "s supports σ", or, "s makes it the case that σ".

The Liar Paradox brings into question three fundamental aspects of logic: truth, negation, and reference. The Liar states a sentence like:

This sentence is not true. Or
What I am saying is false. (1)

If the sentence the Liar has said is true, then it tells us that the sentence is *not* true. This is obviously a contradiction, so as the next logical step, we claim that the sentence the Liar has said is false. But, if this is the case, then the negation makes the sentence read: *This sentence* is *true.* The contradiction goes both ways, and here lies the paradox. What is the real truth value of these kinds of statements? In attempt to solve this problem, many logicians have said that there is a gap in the argument, or that it fails to express any claim. We will examine truth, negation, and reference and their role in this contradiction.

Traditionally, logic has always said that sentences give truth. But is this really the best way to view things? Is truth a property of sentences or things *expressed* when people use sentences? It turns out that mistakes and confusion can result from assigning truth values to sentences. For example, the statement:

Earth is the third planet from the sun. (2)

is one that we all know to be true. Now, if we create a sentence similar in structure to this one, like:

Earl is the third person in the line. (3)

the truth value can either be true or false depending on the situation, and is likely to be true at one moment, and then false the next. According to Russell, sentence (2) would be true. However, sentence (3) it is unclear whether or not the facts are true or not. As a solution to this, Barwise and Etchemendy follow Austin and suggest we make truth a fundamental property of propositions, where a proposition is the claim a sentences makes, not the sentence itself. Propositions can either be true or false, and can be used to make sentences. This distinction is instrumental in the way we attempt to undo this paradox.

Now that we know propositions can be true or false, we can now look at circular propositions. Is a proposition circular because it refers to itself? Reference is the second aspect of the Liar paradox that we need to examine. Barwise and Etchmendy build into both the Russellian and Austinian models “the assumption that one can always use the phrase “this proposition” to refer to something, and that that something can be the very proposition the embedding sentence is used to express.” (Barwise and Etchemendy, pp 15) We have determined that propositions can refer to any propositions, where the words “this proposition” indicate that the statement has the reflexive property.

The final aspect needing clarification is negation. In English, negation can be found in the beginning of a sentence, which negates the whole statement, or in the beginning of a verb phrase. For example:

The cardholder does not have the ace of spades. (4)

It is not the case that the cardholder does have the ace of spades. (5)

In (4), the negation is in the beginning of a verb phrase, where as in (5), the negation applies to the whole statement. In the case of the Liar, the negation is in the beginning of the verb phrase. Now we have clarified our definitions of the terms truth, reference, and negation and how they are applied to propositions. We have also determined that while self reference and negation can be the source of some ambiguities, they are not the cause of this circular argument. That leaves the situation of the proposition to be analyzed.

The statement: “This sentence is not true”, can be expressed using notation from situation theory. Tr is the relation of it being true. The object in question is “this sentence” or p_1. Let s_1 be the situation in which the Liar is true, and referring to p_1. Now we have

$$p_1=\{s_1;[Tr, p_1;0]\}.$$

According to situation theory (Proposition 2 of Modeling Austinian Propositions), p_1 is either true or false, and not both. (Barwise and Etchemendy, pp 127) Let us assume p_1 is true. If p_1 is true, in situation s_1, then it is the case that when the liar is telling the truth, the proposition is true and so “this sentence is not true.” However, there is a contradiction. By the law of excluded middle, we conclude that p1 must then be false. So, assuming we are still in the same situation, we have that when the liar is telling the truth, the proposition is false, so it is not the case that “this sentence is not true”. In other words, it is the case that “this sentence is true.” However, this cannot be because we just assumed that it was false. We reach a contradiction both ways. Barwise and Etchemendy claim that when the proposition is false, the situation changes and it is no longer the case that the liar is referring to a true proposition. Therefore, the act of saying the statement changes the context in which the proposition is in. After realizing this, we no longer have the case in which it is both true and false at the same time.

Situation theory helped to solve the Liar paradox, and many other circular arguments that result from the vagueness of language. The key to understanding the Liar is realizing that there exists a hidden parameter that changes while you reason. For example, a simple paradox can be found in a phone conversation between two people, Person 1 in California, and Person 2 in New York. When the Person 1 states that it is 3 p.m., Person 2 looks at his/her watch, and sees that this is wrong. Person 2 believes it is 6 p.m. Who is right? Here, the hidden parameter is that Person 1 and 2 are not in the same situation. Because each is in a different time zone, the context in which they are telling time is different. Realizing this allows both to understand that they are each correct in their given situation. A common knowledge of these parameters can decrease the amount of confusion, and clarify statements. In mathematics, you rarely see a theorem or proposition stated without first declaring parameters. There are always conditions stated to create a certain set of situations in which the proposition or theorem is true.

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