Sean McSpadden

Bryan Smith

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<center>Descartes, Geometry Parallels Uncanny and Revealing</center>

Philosophy and mathematics have been in human history since its earliest beginnings. Both fulfilled each person's curiosity about the world and yet both were forced to follow very similar paths with modern discoveries. The discovery of non-Euclidean geometry and Descartes' *Meditations on First Philosophy* changed the way mathematicians and philosophers thought about their specialties respectively. These discoveries forced both to break down their structure and build again from the bottom up. It then doesn't seem odd that both Descartes and modern mathematicians use the same method: the axiomatic method to communicate their "ways of knowing." Descartes' dream argument, designed to find the only truths of our world, is in itself an axiomatic system. Descartes' argument follows the most basic steps of "mathematical reasoning" by giving meaning to the pieces of his argument, communicating the argument effectively through proof and discovering irrefutable truths. It is apparent how much his own argument parallels Non-Euclidean Geometry; they both question their own foundations and both seek to find the simplest of truths.

Rene Descartes was born in La Haye, France on March 31, 1596 to a dead mother and a less than friendly father. At ten, he attended a Jesuit college in Anjou where he later learned higher mathematics and philosophy (Cottingham xix). It is ironic that one of the most famous philosophers of all time can also be credited with inventing analytic geometry but it is even more ironic that his three-part *La Geometrie* appeared as an appendix to his famous philosophical

work *Discourse on Method* (Greenberg 34). "Descartes' stated goal was to provide general methods, using algebra, to 'solve any problem in geometry'" (Greenberg 35). This seems to have carried over into his other studies because Descartes' *Meditations on First Philosophy* shook philosophy down to its very foundations, "leav(ing) behind the comfortable world of inherited prejudice and preconceived opinions; tak(ing) nothing for granted in the determination to achieve secure and reliable knowledge" (Cottingham xviii). Even at school, Descartes began to question his world. He "was not impressed with the philosophy he learned at school, and later wrote that the subject, despite being 'cultivated for many centuries by the most excellent of minds', contained no point which was not 'disputed and hence doubtful'. The 'shaky foundations' of the traditional system meant, in his view, that all the specific sciences built on them were equally suspect" (Cottingham xx). At an early age, Descartes was already questioning aspects of philosophy and just like mathematicians post Euclid, he saw that as a dangerous flaw in the entire subject.

After Euclid wrote *Elements*, in which he lays out what most believed to be "Geometry," many mathematicians believed that geometry was a solved problem. After Euclid there was nothing. After all, why not? Everything Euclid published seemed to accurately describe the world around him, not to mention it was highly useful in it as well. However, there was one problem with Euclid's book, one unsolvable problem. "The mystery of why Euclid's parallel postulate could not be proved remained unsolved for more than two thousand years, until the discovery of non-Euclidean geometry [...] revealed the impossibility of any such proof" (Greenberg xiii). Marvin Greenberg, as well as many other mathematicians, believed "this discovery shattered the traditional conception of geometry as the true description of physical space" (Greenberg xiii). Since geometry, in essence, could not accurately describe physical

space, this destroyed any notion that anything in geometry was accurate unless inside the constraints of Euclid. So, just like Descartes, mathematicians began from the beginning and attempted to "reprove" much of geometry, following an exact system in which only pure logic could prove what they already thought they knew.

Descartes' *Meditations on First Philosophy*, a narrative on his troubling thoughts about the world, is his form of "reproving" his field of philosophy. Descartes, remembering that he believed in false things as a child, questions how much the human race takes for granted about the "truths" of life. He realized "that it was necessary, once in the course of (his) life, to demolish everything completely and start again right from the foundations if (he) wanted to establish anything at all in the sciences that was stable and likely to last" (Descartes 12). By questioning the very core of every human presumption, Descartes makes some shocking, yet viable discoveries.

In his first two meditations, Descartes creates a "dream argument" that allows him to question even the very existence of the world around him. Through his dream argument, Descartes eliminates the potential lies society has accepted and discovers the few truths left which provides him with a new foundation to re-evaluate his life. This argument is a complex one and cannot be understood unless put into context. Descartes begins his meditation by announcing the possibility that human beings could be wrong about everything they once took for granted. His goal is to "clean house" essentially and start the foundations of his life on undeniable truths because "once the foundations of a building are undermined, anything built on them collapses of its own accord; so I will go straight for the basic principles on which all my former beliefs rested" (Descartes 12). He attempts to do this by finding anything that could have any remote chance of being false. He does this in order to insure that he may find the few

undeniable truths that remain. He discounts many ideas people normally accept as true through this argument.

In the dream argument, he questions the validity of the reality he resides in. Most people assume that they have hands, have feet etc. However, he is able to discredit a great deal of this. His dream argument is as follows: a man believes himself to be sitting in his room watching the television. A moment later the alarm goes off and he realizes he had dreamt that entire scene. He quotes "How often, asleep at night, am I convinced of just such familiar events—that I am here in my dressing-gown, sitting by the fire—when in fact I am lying undressed in bed!" (Descartes 13). Descartes argues that it is possible that the reality he believes himself to be a part of (the one of him waking to the alarm clock), could also be a dream world, and he has yet to wake up to true reality. Hence, the fact that he has hands and is sitting in his chair could be discredited, as this whole scene could be a figment of his imagination. Through this argument, Descartes is able to discard almost everything he learned in this possibly fake world. Relating back to his dream argument he states, "Suppose then that I am dreaming, and that these particulars—that my eyes are open, that I am moving my head and stretching out my hands—are not true. Perhaps, indeed, I do not even have such hands or such a body at all" (Descartes 13). By saying it is possible he has lived in a dream world his whole life, he discredits anything he has learned inside of this dream world.

The truths that Descartes discovered (which will be explained later) have a remarkable parallel to the very foundations of Geometry. Both Descartes and Non-Euclidean geometry put particular emphasis on the definitions of the "subjects" they work with. In geometry, definitions are used to define a figure in its most basic form so that, when it is needed in proofs, it cannot be judged in any other way than the intended one. For example, three or more points are **collinear**

if there exists a line incident with all of them. This definition is simple yet no one is left in any

doubt about what collinear means. Descartes also stresses the great importance of correct

definitions. He enumerates on how the dangers of incorrect definitions can destroy any work. He

states that when one thinks of candle wax, he may define it as flowery smelling with a solid and

sturdy shape, hard and cold. But Descartes points out that these things change if the candle wax

begins to melt and everything that was once defined correctly now is inadequate in describing

the wax. Descartes comments:

> **"The wax was not after all the sweetness of the honey, or the fragrance of the flowers, or the whiteness, or the shape, or the sound, but was rather a body which presented itself to me in these various forms a little while ago, but which no exhibits different ones. But what exactly is it that I am now imagining? Let us concentrate, take away everything which does not belong to the wax, and see what is left: merely something extended, flexible changeable." (Descartes 20)**

These last three words extended, flexible (not in the sense of one form, but between the two

forms), and changeable turn out to be the definition of wax because those three words are the

only ones able to describe wax in any situation. By giving express definitions, Descartes was

better able to find his undeniable truths.

Descartes' truths, almost directly correlate to geometric axioms. Axioms are the basis of

Euclidean and non-Euclidean geometry. They are statements that are improvable but seem to be

inherently true. In formal argument, Marvin Greenberg harkens it to a statement "that you

already accept, one that I do not need to justify" (Greenberg 10). For example, in incidence

geometry its axioms are:

1.) For every point P and every point Q not equal to P, there exists a unique line that

passes through P and Q.

2.) For every line there exists at least two points incident on it.

3.) There exists three distinct points such that no line is incident on all three.

As seen in Incidence Axiom 1, there is no "reason" why a line forms between P and Q it just happens that everyone believes it to be true.

The importance of axioms is that they do not allow any assumptions to be made that aren't expressly written in or proven from the axioms. This is important because it makes it impossible to derive anything true from diagrams (as had been accepted for so long in Geometry) because they can possibly mislead a person. Greenberg states, "a diagram may be slightly inaccurate or it may represent only a special case. If we are to recognize the flaws in arguments, […] we must not be misled by diagrams that *look* plausible" (Greenberg 25).

Descartes expressly agrees on this point because he states that its assumptions that have led human beings astray. Once again, his dream argument throws these assumptions out of the window. He reminds readers that when they dream they believe their dream to be entirely real, but when they wake up they immediately discount the dream's validity. Through this argument, he says its possible that perhaps there is an all powerful being, with "none of God's goodness", that could be deceiving the world into thinking that what they were experiencing was real—when in fact they are living in the dream world the demon created. By stating this he calls into question even the simple ideas of colors and mathematics. He states, "How do I know that he (the demon) has not brought it about that there is no earth, no sky, no extended thing […] How do I know that God has not brought it about that I too go wrong every time I add two and three and count the sides of a square?" (Descartes 14). By assuming the worst possible situation—that a greater being is deceiving him—he is forced to accept that even colors and (interestingly enough) mathematics cannot be fully trustworthy.

The parallel between geometry and Descartes arguments is that they both follow the axiomatic method. The axiomatic method is "a method of proving that the results are correct and

organizing them into a logical structure" in order to communicate what the mathematician has done (Greenberg 9). Descartes employs this "mathematical reasoning" because he follows the method in its entirety. The first requirement of the axiomatic method is that everyone has "a mutual understanding of the meaning of the words and symbols used in the discourse" (Greenberg 11). This is easy enough with easily defined terms such as the ones previously mentioned. However, there are undefined terms in both geometry and philosophy. Point and line are undefined terms whereas Descartes struggles to define I, and existence. Although he cannot define them, everyone has a basic enough understanding of what these words mean so Descartes uses that model of his axiomatic system to explain his ideas.

The second requirement of Descartes is that he has to have people agree on "how and when one statement follows logically from another" (Greenberg 10). At this point, it is important to note that Descartes works in reverse of the actual axiomatic system, finding the simplest correct answer to disprove the wrong ones, because he breaks down every possible wrong answer to find the correct instead. Descartes uses his on variation of proof by cases in order to discover his truths. Descartes essentially eliminates all possibilities on potential truths. For example, he discounts anything received from the senses: "whatever I have up till now accepted as most true I have acquired either from the senses or through the senses. But from time to time I have found that the senses deceive, and it is prudent never to trust completely those who have deceived us even once" (Descartes 12). He continues this over and over again, discounting anything with a shred of doubt until at last he finds his three truths, his three axioms.

Descartes' axioms are undeniable because they are so simple. At last, at the end of the Second Meditation, Descartes' dream argument is unable to discount the three's validity. He gives his first axiom as his first building block: "So after considering everything very

thoroughly, I must conclude that this proposition, *I am, I exist*, is necessarily true whenever it is put forward by me or conceived in my mind" (Descartes 17). To put it simply, his ability to conceive that he may not exist proves that he exists. Where else would his thoughts come from? If he did not exist, he would lack the necessary thought processes to understand existence.

These thought processes are Descartes second truth. He brings to light that at the very least he thinks. In his own words, he is a "thing that thinks" (Descartes 19). As defined a thing is "anything that is or may become an object of thought" (Dictionary). Most of Descartes thoughts in his Meditations surround himself, and this self-reflection, proven by definition, shows that he is a thing.

This final truth is that he is a thinker who thinks about the many various experiences he has lived. He recognizes that although he doesn't *know* he is sitting in his chair, he at least thinks he is sitting in one. This thought is undeniable. The chair may be false in reality, but Descartes' thought that it is real is a genuine thought. This combined with the other two other axioms give Descartes his foundation—he exists as a thinker that thinks about the many things in his world, regardless if they are true or false themselves.

His set of axioms would look like:

Descartes Axiom 1: He exists

Descartes Axiom 2: He thinks

Descartes Axiom 3: If he thinks, then they are about his experiences.

Through his dream argument Descartes is able to whittle away all the potential farces he has lived and is able to find these three unarguable truths. His argument, although strange, is almost impossible to refute. No potential story is able to rebuff that Descartes exists as a thinker that thinks. This is, perhaps, his most important argument in *Meditations on First Philosophy*

simply because he could not proceed forward in his theories without having the truths he discovered as a base. It is now the same for Geometry. The discovery of non-Euclidean geometries straightened out the correct path for mathematics to travel. By following the axiomatic system, mathematicians are finding indisputable truths that are all proven by structured logic.

Whether he concentrated on mathematics or philosophy, Descartes can always be connected to geometry forever simply because of his rigid axiomatic structure and his uncanny parallels to geometric history. Descartes' theories question every aspect of society but also provide some concrete truths that help strengthen it; non-Euclidean geometry has thrown all of geometry into question but it is now correct and irrefutable. Without the dream argument, Descartes as well as mathematicians would have been unable to discover the things they did, and without those discoveries the rest of society may still have lived under its potentially false delusions.

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