Mathematics 181 – B

Fall 2007

Exam 4

November 20, 2007

____
Name

Technology used: ______________________________

Directions: Be sure to include in-line citations every time you use technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

Evaluate each integral in this exam BY HAND unless told otherwise.

Required Problem (10 points) Use the definition of limit of a sequence (this requires you use "ε") to **prove** that $\lim_{n\to\infty} \frac{n}{n+1} = 1$.

Do any three (2) of the following.

1. (15 points) Find the smallest value of n so that the error in using Simpson's Rule to approximate $\int_1^3 \frac{1}{x}dx = \ln(3)$ is less than 10^{-3}.
2. (15 points) Evaluate the following improper integral.

$$\int_0^1 \frac{1}{\sqrt{x}\,(x+1)}dx$$

3. (15 points) How many integrals with a single impropriety at a limit of integration are required to evaluate $\int_{-\infty}^{\infty} \frac{x}{(x-1)(x+4)}dx$? Give reasons for your answer.

Do any three (3) of the following

1. (20 points) Determine whether the infinite series $\frac{1}{3} + \frac{2}{7} + \frac{3}{11} + \frac{4}{15} + \frac{5}{19} + \cdots$ converges or diverges. [Hint: First write the series using "$\sum$" notation.]
2. (20 points) Do **one** (1) of the following.
   (a) Find the sum of this convergent geometric series. $\sum_{n=3}^{\infty} \frac{1}{11}\left(-\frac{7}{9}\right)^{n-2}$. Do **not** give the answer in decimal form.
   (b) Find the sum of the telescoping series $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$.
3. (20 points) Explain, briefly and in your own words,
   (a) Why is it true that if $\sum_{n=1}^{\infty} a_n$ is a divergent **series** of positive numbers then there is also a divergent series $\sum_{n=1}^{\infty} b_n$ of positive numbers with $b_n < a_n$ for every n?
   (b) Is there a "smallest" divergent series of positive numbers?
4. (20 points) Select **two** (2) of the following infinite series and determine if they converge or diverge. Indicate which test you are using and show all work justifying your conclusion.
   (a) $\sum_{n=2}^{\infty} \frac{1}{[\ln(n)]^2}$

(b) $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n^2+1}$

(c) $\sum_{n=2}^{\infty} \frac{[\ln(n)]^2}{n^2}$