Mathematics 181–C Spring 2007

Exam 4

April 24, 2007

_____
Name

Technology used: _____ Directions:

- Be sure to include in-line citations every time you use technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page. When given a choice, specify which problem(s) you wish graded.**

The Problems

1. [15 points] Find the exact sum of **one** (1) of the following converging series.

   (a) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$ [Use partial fractions.]

   (b) $\sum_{n=5}^{\infty} 3\left(\frac{1}{4}\right)^{n-1}$ [This geometric series in **not** in standard form.]

2. [10 points each] Determine the divergence or convergence of any **four** (4) of the following infinite series. **State the test you use and show all necessary work.**

   (a) $\sum_{n=1}^{\infty} \left(1+\frac{7}{n}\right)^n$

   (b) $\sum_{n=1}^{\infty} \frac{1}{3^{n-1}-1}$

   (c)
   $$\sum_{n=1}^{\infty} \frac{1}{1+2^2+3^2+\cdots+n^2}$$

   (d) [Be careful with the factorials.]
   $$\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$$

   (e) [If this series converges, determine if it does so absolutely or conditionally.]
   $$\sum_{n=1}^{\infty} (-1)^n \frac{1}{1+\sqrt{n}}$$

3. Do any **three** (3) of the following.

   (a) Find the radius and interval of convergence for the following series. For what values of x does the series converge absolutely and for what values conditionally?
   $$\sum_{n=0}^{\infty} (-2)^n (n+1)(x-1)^n$$

   (b) Find the Taylor series generated by the function $f(x) = x^4 + x^2 + 1$ at $a = -2$.

(c) Find a polynomial that will approximate the function $F(x)$ given below throughout the interval $[0, 1]$ with an error of magnitude less than 10^{-6}.

$$F(x) = \int_0^x t^2 e^{-t^2} \, dt$$

(d) Use the **binomial series** and the fact that $\frac{d}{dx}[\arcsin(x)] = (1 - x^2)^{-\frac{1}{2}}$ to generate the first four **non-zero** terms of the Taylor series for $\arcsin(x)$. What is the radius of convergence?