Mathematics 181A — Fall 2008

Exam 1

September 18, 2008

_____
Name

Technology used: _____ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

1. [10 points] Do **one** (1) of the following.

1. (a) A point P in the first quadrant lies on the graph of the function $f(x) = \sqrt[3]{x}$. Express the x-coordinate of P as a function of the slope of the line joining P to the origin.

   (b) If a composite $f \circ g$ is one-to-one, must g be one-to-one? Explain your answer.

2. [15 points] Rewrite the following sum as indicated.

$$\sum_{k=4}^{101} (2k-1)^2 = \sum_{j=15}$$

3. [15 points] Do **one** (1) of the following. Show your work.

1. (a) Evaluate $\int \frac{1}{t^3}\left(t^2 - 3t^5 + t^{1/2} + 5t^3\sec^2(t) + 6t^3\sec(t)\tan(t) + \frac{t^3}{\sqrt{1-t^2}}\right)\,dt$

   (b) By differentiating the right hand side, verify the formula $\int \frac{\arctan(x)}{x^2}\,dx = \ln(x) - \frac{1}{2}\ln(1+x^2) - \frac{\arctan(x)}{x} + C$

4. [8, 7 points] The following is a Riemann sum for a function f with domain an interval $[a, b]$. [Do NOT simplify this sum.]

$$\sum_{k=1}^{n} \left[3\left(5 + \frac{6k}{n}\right)^7 - \left(5 + \frac{6k}{n}\right)^2 + 6 \right] \frac{6}{n}.$$

1. (a) i. What is this specific $f(x)$?

   ii. What is the specific interval $[a, b]$?

5. [15 points] Find the derivative of $G(x) = \int_{x^4}^{x} e^{t^2}\ dt$ using part 1 of the Fundamental Theorem of Calculus.

6. [15 points each] Do **both** of the following.

1. (a) Evaluate $\int (2t + 1 + 2\cos(2t + 1))\ dt$

   (b) Evaluate $\int \frac{(\ln(x+1))^2}{x+1}\ dx$