Mathematics 181D — Spring 2008

Exam 1

February 7, 2008

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Name

Technology used: _____ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

Do both of these problems

1. Find the derivative of $F(x) = \int_x^{e^x} t^2 \, dt$
   (a) Using part 1 of the Fundamental Theorem of Calculus
   (b) Using part 2 of the Fundamental Theorem of Calculus
2. Use substitution to evaluate two of the following indefinite integrals.
   (a) $\int \frac{1}{t^3} \sin\left(3 - \frac{1}{t^2}\right) dt$
   (b) $\int \frac{\sqrt{3+\arctan(x)}}{1+x^2} dx$
   (c) $\int \sqrt{1-x^2} dx$ start by using the substitution $x = \sin(\theta)$.

Do any four (4) of the following problems

1. (8, 7 points) Solve the initial value problem
$$\frac{d^3y}{dx^3} = x, \quad \left.\frac{d^2y}{dx^2}\right|_{x=0} = 2, \quad y'(0) = 3, \quad y(0) = 1$$
2. Without using a calculator, evaluate **both** of the following indefinite integrals
   (a) $\int \left(7\sec^2(x) - \frac{2}{1+x^2} + \sec(x)\tan(x) + \frac{1}{x^{3/4}}\right) dx$
   (b) $\int \frac{1}{y^2}\left(2y^3 + 3y^2 + 4y + y^{1/2}\right) dy$
3. If we use the partition points $x_0 < x_1 < x_2 < \cdots < x_n$ to partition the interval $[1, 5]$ into n subintervals of equal length.
   (a) What is the value of Δx in terms of the letter n?
   (b) Write the values of x_0, x_1, x_2, x_k, and x_n in terms of the letter n.
   (c) Use sigma notation to write, in terms of the letter n, the Riemann sum for the function $f(x) = x - x^2$ that uses the left endpoint of each subinterval as the value of c_k.**Do not simplify this Riemann Sum.**
4. If we partition the interval $[0, 3]$ into n subintervals of equal width, then the Riemann sum for the function $f(x) = 4x - x^3$ that uses this partition and the right endpoint of each subinterval as the value of c_k is $\sum_{k=1}^{n} \left[4\left(0 + \frac{3k}{n}\right) - \left(0 + \frac{3k}{n}\right)^3\right] \frac{3}{n}$.

(a) Use limits to compute the value of $\int_0^3 \left(4x - x^3\right)\ dx$. [No credit if you use the Fundamental Theorem of Calculus.]
Useful facts: $\sum_{k=1}^n k = \frac{1}{2}n\left(n+1\right), \quad \sum_{k=1}^n k^3 = \frac{1}{4}n^2\left(n+1\right)^2$.

5. Given the function $f\left(x\right) = \sqrt{x^2+1}$ with domain the interval $\left[0,5\right]$. Write a Riemann sum for f using a partition P that divides $[0,5]$ into 3 subintervals and where $\|P\| = 2$. Be sure to specify the partition points of P as well as writing out the Riemann Sum **without** using sigma notation.

6. Suppose that f and g are integrable functions and that $\int_a^b \left(3f\left(x\right) - g\left(x\right)\right)\ dx = 5$ and $\int_a^b \left(f\left(x\right) + g\left(x\right)\right)\ dx = 7$. Use properties of definite integrals to find $\int_a^b f\left(x\right)\ dx$ and $\int_a^b g\left(x\right)\ dx$.