Mathematics 181A **Fall 2008**

Exam 4

December 4, 2008

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Name

Technology used: ____ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

1. [5, 5 points] The Taylor Series generated by a certain function f at $x = 2$ is $\sum_{n=0}^{\infty} \frac{2^n}{(2n+1)} (x-2)^{2n+1}$.

   (a) Determine $f^{(5)}(2)$.

   (b) Determine $f^{(100)}(2)$.

2. [15 points] Do **one** (1) of the following.

   (a) Find the Taylor Series generated by $f(x) = 2x^3 - x + 3$ at $x = 2$.

   (b) Use sigma notation to write the quadratic approximation for the function $f(x) = (2-x)^{-2}$ at $x = 1$.

3. [15 points each] Do two of the following.

Which of the following series converge and which diverge? Is the convergence conditional or absolute?

(a) $\sum_{n=1}^{\infty} a_n$ where $a_n = \frac{(2n)!}{(n+1)!(n+2)!}$

(b) $\sum_{n=1}^{\infty} \frac{n^{13}}{5^n}$

(c) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt[3]{n}}$

4. [15 points] Find the center, radius of convergence, and interval of convergence for the following series. Specify the values of x for which the series converges absolutely and the values for which it converges conditionally.

$$\sum_{n=1}^{\infty} \frac{2\,(n+1)}{5^n n^2}(x+1)^n$$

5. [15 points] Find the Taylor Series generated by $f(x) = \ln(x)$ at $x = 1$. Write your answer in sigma notation.

6. [15 points] The series $\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$ and $\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$ converge to $\cos(x)$ and $\sin(x)$, respectively for all values of x. Use series multiplication to find the first four non-zero terms of a series for $\cos(x)\sin(x)$.