Mathematics 181B Spring 2012

Exam 4

April 26, 2012

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Name

Technology used: ______________________________________ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

"A chief event of life is the day in which we have encountered a mind that startled us". -Ralph Waldo Emerson, writer and philosopher (1803-1882)

Problems

1. [15 points] From the quiz: Find a polynomial that will approximate the following function throughout the interval $[0, 0.5]$ with an error of magnitude less than 10^{-3}

$$F(x) = \int_0^x \arctan(t)\ dt$$

2. [15 points] Using any results from Chapter 8, build the Maclaurin Series for the following function. [Exploiting known Taylor Series is the easiest approach.]

$$f(x) = \frac{x^3}{1-3x}$$

3. [10 points each] Do any **two** (2) of the following.

   Use appropriate convergence tests to determine whether the following series of positive terms converge or diverge. Show your work.

   (a) $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}+\sqrt[3]{n}}$

   (b) $\sum_{n=1}^{\infty} \left(1 - \frac{1/3}{n}\right)^n$

   (c) $\sum_{n=1}^{\infty} \frac{(10{,}000)^n}{n!}$

   (d) $\sum_{n=1}^{\infty} \frac{1}{1+2+3+\cdots+n}$

4. [15 points] Use an appropriate convergence test to determine if the following series of positive terms converges

$$\sum_{n=0}^{\infty} \frac{n!\,(n+1)!\,(n+2)!}{(3n)!}$$

5. [15 points] Determine the radius of convergence, interval of convergence and numbers where convergence is conditional for the power series

$$\sum_{n=0}^{\infty} \frac{(4x-5)^{n+1}}{2n}$$

6. [10 points] Prove the theorem that absolute convergence implies convergence. More specifically, **prove** that if the series $\sum_{n=1}^{\infty} |a_n|$ converges then so does the series $\sum_{n=1}^{\infty} a_n$.

7. [10 points] Replace the following equation with an equivalent polar equation.

$$(x+2)^2 + (y-5)^2 = 16$$

8. **Extra Credit**. [5 points] Give an example of converging series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ for which the series $\sum_{n=1}^{\infty} a_n b_n$ diverges. [Note that the terms do not need to be positive.]

Useful Information

- The Taylor series generated by the function f at $x = a$ is

$$\sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(a)(x-a)^n$$

and the remainder term is

$$R_n(x) = \frac{1}{(n+1)!} f^{(n+1)}(c)(x-a)^{n+1}$$

where c is a number between a and x.

- If $\sum_{n=0}^{\infty} (-1)^n u_n$ is a convergent alternating series with sum S and if

$$\begin{aligned} s_n &= \sum_{k=0}^{n} (-1)^k u_k \\ &= u_0 - u_1 + u_2 - u_3 + \cdots + (-1)^n u_n \end{aligned}$$

is the n'th partial sum of the original series, then the n'th partial sum approximates the exact sum to within u_{n+1}. That is, $|S - s_n| < u_{n+1}$.

- The Maclaurin series for the arctangent function is

$$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \qquad |x| \leq 1.$$