Mathematics 181A **Fall 2010**

Exam 5

December 2, 2010

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Name

Technology used: ____ **Only write on one side of each page.**
Show all of your work. Calculators may be used for numerical calculations and answer checking only.

Do BOTH of these problems

1. (20 points) Do all the work to find the Taylor series generated by $f(x) = \frac{1}{x^2}$ at $x = 1$. (See the Useful Information at the end of this exam.)

2. $(10, 5, 5$ points) For the power series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(3x-1)^n}{n}$$

   (a) Determine the radius and interval of convergence.
   (b) Determine the numbers x where the series converges absolutely.
   (c) Determine the numbers x where the series converges conditionally.

Do any three (3) of the following problems

3. $(10, 10$ points) Because it is a geometric series, we know that the infinite series $f(x) = \frac{1}{1-x} = \sum_{k=0}^{\infty} x^n = 1 + x + x^2 + x^3 \cdots$ converges if and only if $-1 < x < 1$.

   (a) Take the term by term derivative of $f(x)$ and write it in sigma notation as well as in "dot, dot, dot $(\cdots)$" notation.
   (b) Find the first four terms (a_0, a_1, a_2, a_3) of the power series for $\left(\frac{1}{1-x}\right)^2$ by multiplying the power series of $\frac{1}{1-x}$ times itself as indicated below. $(1 + x + x^2 + x^3 + \cdots)(1 + x + x^2 + x^3 + \cdots) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$.

4. $(10, 10$ points)

   (a) The first few terms of the Taylor series at $x = 2$ for a function f are $f(x) = 12 - 4(x-2) + 25(x-2)^2 - 6(x-2)^5 + 5(x-2)^6 + \cdots$. What are $f^{(5)}(2)$ and $f^{(4)}(2)$?
   (b) Suppose we know that the infinite series $\sum_{n=1}^{\infty} c_n (x-3)^n$ converges at the value $x = 4$. Give three other values of x at which the series converges and briefly explain how you know it converges at those points.

5. $(10, 5, 5$ points) We know that $\int_0^x \frac{1}{1-t} dt = -\ln|1-x|$ and that $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^n$ for all x satisfying $|x| < 1$.

   (a) Use this and term-by-term integration to find the Taylor series for $\ln|1-x|$
   (b) Use the Alternating Series Test to show that this integral series converges at $x = -1$.

(c) Use this information to determine the exact sum of the alternating harmonic series.

6. (20 points) Determine if the following series, diverges, converges absolutely, or converges conditionally.

$$\sum_{n=1}^{\infty} (-1)^n n^2 \left(\frac{2}{3}\right)^n$$

7. (20 points) It can be shown (but you do not need to do it) that the series $\sum_{k=1}^{\infty} \frac{(-1)^k}{\ln(k+1)}$ is a convergent alternating series. Use the error bound formula for alternating series (see the Useful Information at the end of this exam) to determine a value of n that guarantees that the n'th partial sum $S_n = \sum_{k=1}^{n} \frac{(-1)^k}{\ln(k+1)}$ of this series is accurate to within 10^{-2}. How does your calculator represent this number? (This should strike you as 'slow convergence' of a series.)

Useful Information

1. The Taylor series generated by the f at $x = a$ is

$$\sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(a)(x-a)^n$$

2. If $\sum_{n=0}^{\infty} (-1)^n u_n$ is a convergent alternating series with sum S and if

$$\begin{aligned} s_n &= \sum_{k=0}^{n} (-1)^k u_k \\ &= u_0 - u_1 + u_2 - u_3 + \cdots + (-1)^n u_n \end{aligned}$$

is the n'th partial sum of the original series, then the n'th partial sum approximates the exact sum to within u_{n+1}. That is, $|S - s_n| < u_{n+1}$.