Mathematics 181D **Spring 2008**

Final Examination

May 12, 2008

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Name

Technology used: ____ **Only write on one side of each page.**

Show all of your work. Calculators may be used for numerical calculations and answer checking only.

1. Do **one** (1) of the following.

   (a) Find $\frac{dy}{dx}$ for the following function

   $$y = G(x) = \int_{x^2}^{\arcsin(x)} e^{\left(t^4\right)}\, dt$$

   (b) Solve the following equation for $f(x)$.

   $$\cos\left(x^3\right) + 2 = \int_2^{x^2} t f(t)\ dt$$

2. Determine whether the following improper integral converges or diverges. If it converges, find its value.

   $$\int_0^1 \frac{e^x}{(e^x - 1)^{1/2}}\, dx$$

3. Without using the integral tables or your calculator, evaluate **four** (4) of the following integrals.

   (a)

   $$\int \sin^4(x)\cos^5(x)\, dx$$

   (b)

   $$\int \frac{x^3 + 3x^2 - 26x - 41}{x^2 + 3x - 28}\, dx$$

   (c)

   $$\int \frac{dt}{\sqrt{2x - x^2}} \quad \text{HInt: complete the square}$$

   (d)

   $$\int \frac{e^{2x} dx}{1 + e^{4x}}$$

   (e)

   $$\int \frac{e^{3x} dx}{\sqrt{1 + e^{3x}}}\, dx$$

4. A hot object is plunged into a beaker of ice water maintained at $40°F$. One and two minutes later, the temperature of the object is measured to be $87°F$ and $76°F$, respectively. Use Newton's Law of Cooling to determine the temperature of the object just before it was immersed (i.e., the initial temperature)?

5. For **three** (3) of the following: Determine if the following series are convergent or divergent. If appropriate, determine if the convergence is absolute of conditional. Be sure to specify which tests you use.

   (a)
$$\sum_{n=1}^{\infty} \frac{\cos(n)}{2^n}$$

   (b)
$$\sum_{k=1}^{\infty} (-1)^k \left(\frac{1+k^2}{k^2}\right)$$

   (c)
$$\sum_{n=1}^{\infty} \frac{1}{\ln(n)}$$

   (d)
$$\sum_{n=1}^{\infty} (-1)^k \left(\frac{1+k^2}{k^3}\right)$$

6. Find all numbers x at which **one** (1) of the following power series converges absolutely and all numbers where it converges conditionally. Explain your answer.

   (a)
$$\sum_{n=0}^{\infty} (-1)^n \frac{(2x-3)^n}{n}$$

   (b)
$$\sum_{n=1}^{\infty} \frac{2 \cdot 5 \cdot 8 \cdot \cdots \cdot (3n-1)}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)} x^n$$

7. Do **one** (1) of the following.

   (a) Suppose we know that the infinite series $\sum_{n=1}^{\infty} c_n(x-2)^n$ converges at the value $x = 5$. Do we have enough information to know if the series converges at the value $x = 0$? Why or why not?

   (b) Suppose $P_8(x) = 5 - 2(x-2) + 4(x-2)^2 - 3(x-2)^7 + (x-2)^8$ is the eighth-degree Taylor Polynomial for the function $y = f(x)$ at $x = 2$. What are the numerical values of $f'''(2)$ and $f^{(7)}(2)$?

8. Starting with the Taylor series for $f(x) = \frac{1}{1-x}$, estimate the following integral to within 0.01. Be sure to justify that your answer is as accurate as claimed. You may use your calculator only for computing sums, differences, products and quotients.
$$\int_0^1 \frac{x}{1+x^3}\, dx$$

9. Do **one** (1) of the following.

(a) Find the area of the region enclosed by one leaf of the three-leaved rose $r = \sin(3\theta)$

(b) Find the length of the curve given by the polar coordinate equation $r = 8\sin^3(\theta/3)$, $0 \leq \theta \leq \pi/4$