Mathematics 290-A Exam 2 Fall 2006

October 3, 2006

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Name

Technology used: ______________________

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**
- **When given a choice, be sure to specify which problem(s) you want graded.**

Do any three (3) of these computational problems

C.1. Do all of the following.

(a) Show that the set of vectors $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} \right\}$ is linearly dependent.

(b) Find two vectors $\vec{w}_1, \vec{w}_2$ that are both in S and for which $< S > = < T >$, where $T = \{\vec{w}_1, \vec{w}_2\}$.

(c) Write one of the extra vectors in S as a linear combination of $\vec{w}_1$, and $\vec{w}_2$.

C.2. Write all of the following complex numbers in the form $a + bi$.

(a) $2(2-3i) - 7(6+2i)$

(b) $\frac{4+3i}{2-i}$

(c) $\sqrt{i}$ [Hint: write $(a+bi)^2 = i$ and solve a system of equations.]

C.3. The vectors $\vec{u}_1, \vec{u}_2$,and $\vec{u}_3$ below form an orthonormal set. Use the Gram-Schmidt procedure to find a vector $\vec{u}_4$ so that $\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\}$ is an **orthonormal** set which has the same span as $\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{v}_4\}$.

$$\vec{u}_1 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The Gram-Schmidt formula is

$$\vec{u}_i = \vec{v}_i - \left(\frac{< \vec{v}_i, \vec{u}_1 >}{< \vec{u}_1, \vec{u}_1 >} \right) \vec{u}_1 - \cdots - \left(\frac{< \vec{v}_i, \vec{u}_{i-1} >}{< \vec{u}_{i-1}, \vec{u}_{i-1} >} \right) \vec{u}_{i-1}$$

C.4. Compute the following matrix-vector product **by hand** in two ways.

$$\begin{bmatrix} 1 & 1 & 1 \\ -4 & 1 & 1 \\ 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix}.$$

Do any two (2) of these problems from the text, homework, or class.

You may NOT just cite a theorem or result in the text. You must prove these results.

M.1. Suppose $S = \{\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_p\}$ is a linearly independent set and that $\mathbf{v} \notin \langle S \rangle$. Prove the set $W = \{\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_p, \mathbf{v}\}$ is a linearly independent set.

M.2. Suppose $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a linearly independent set in R^5. Is the set of vectors $2\vec{v}_1 + \vec{v}_2 + 3\vec{v}_3,\ \vec{v}_2 + 5\vec{v}_3,\ 3\vec{v}_1 + \vec{v}_2 + 2\vec{v}_3$ linearly dependent or independent?

M.3. Prove Theorem TMA, Transpose and Matrix Addition.

Suppose that A and B are $m \times n$ matrices. Then $(A + B)^t = A^t + B^t$.

Do one (1) of these problems you've not seen before.

T.1. Suppose A is a square matrix of size n satisfying $A^2 = AA = O$. Prove that the only vector $\vec{x}$ satisfying $(I_n - A)\,\vec{x} = \vec{0}$ is the zero vector.

T.2. Recall that $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Now explain why the fact that $\begin{bmatrix} 3 & 2 & 0 & 1 & 0 & 0 \\ -4 & -2 & -2 & 0 & 1 & 0 \\ -5 & -2 & -4 & 0 & 0 & 1 \end{bmatrix}$ has reduced row-echelon form $\begin{bmatrix} 1 & 0 & 2 & 0 & 1 & -1 \\ 0 & 1 & -3 & 0 & -\frac{5}{2} & 2 \\ 0 & 0 & 0 & 1 & 2 & -1 \end{bmatrix}$ tells us the only vectors $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ that can be in the span of $S = \left\{ \begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ -4 \end{bmatrix} \right\}$ are those where $a + 2b - c = 0$.