Mathematics 290-A **Exam 3** **Fall 2006**

October 24, 2006

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Name

Technology used: ____

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**
- **When given a choice, be sure to specify which problem(s) you want graded.**

Do any two (2) of these computational problems

C.1. Given matrix

$$A = \begin{bmatrix} 1 & 2 & -4 & 3 \\ 2 & 4 & -8 & 6 \\ 3 & 6 & -12 & 9 \end{bmatrix}$$

(a) Use the fact that $C(A) = R(A^t)$ to find a set S where $< S >= C(A)$.

(b) Use the fact that $C(A) = N(L)$ to find a set T where $< T >= C(A)$.

C.2. Do **one** (1) of the following:

(a) A square matrix A is defined to be **skew-symmetric** if $A^t = -A$. [Note that this forces the elements on the main diagonal to be zero.]

Show that the set W of skew-symmetric 3×3 matrices is a subspace of $M_{3,3}$.

(b) An infinite sequence is said to be a **geometric sequence** if it has the form $(a, ar, ar^2, ar^3, \cdots)$ for some complex numbers a and r.

Show that the set W of all geometric sequences is **NOT** a subspace of the vector space C^∞ of all infinite sequences with complex entries.

C.3. Compute the inverse of the following matrix or determine that it is not invertible **by hand.**

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & -1 & 0 & 0 \\ 2 & 2 & 5 & 4 \\ 0 & 3 & 0 & 1 \end{bmatrix}$$

Do any two (2) of these problems from the text, homework, or class.

You may NOT just cite a theorem or result in the text. You must prove these results.

M.1. Prove Theorem **AIU,** Additive Inverses are Unique:

Suppose that V is a vector space. For each $u \in V$, the additive inverse, $-u$, is unique.

M.2. Prove that if A is a square matrix where $N(A^2) = N(A^3)$ (the null space of A^2 is the same set as the null space of A^3) then $N(A^4) \subset N(A^3)$.

M.3. Find a spanning set, S, for the vector subspace W of $M_{3,3}$ where $W = \left\{A \in M_{3,3} : A^t = -A\right\}$ of skew-symmetric 3×3 matrices. [You have not seen this problem before but it is very similar to examples done in class.]

Do two (2) of these problems you've not seen before.

T.1. Prove that if A is $n \times m$, B is $m \times p$ and $AB = O_{n\times p}$ then $C(B) \subset N(A)$.That is, the column space of B is a subset of the Null space of A.

T.2. Given a square matrix A, prove **both** of the following.

(a) $N(A) \subset N\left(A^2\right)$

(b) $C\left(A^2\right) \subset C(A)$

T.3. Suppose V is a vector space, α is a complex number and W is a subspace of V. Show that the set $\alpha W = \{\alpha\vec{w} : \vec{w} \in W\}$ is a subspace of V.