Mathematics 434-A **Spring 2002**

Exam 1

March 28, 2002

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Name

Textbook/Notes used: ______________________________

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. **Only write on one side of each page.**

Do any five (5) of the following.

1. Let $\phi : \mathbf{Z}[x] \to \mathbf{R}$ be the ring homomorphism given by $\phi(f(x)) = f\left(1+\sqrt{2}\right)$. Show that $\ker(\phi) = (x^2 - 2x - 1)$ the ideal generated by $x^2 - 2x - 1$.

2. There are $70 = \binom{8}{4}$ ways to color the edges of an octogon, in which exactly four of the edges are black and the other four are white. The group D_8 operates on this set of 70 colorings and the orbits represent equivalent colorings. Use Burnside's Theorem to count the number of equivalence classes of colorings.

3. Do **both** of the following.

   (a) The set $\{1, 9, 16, 22, 29, 53, 74, 79, 81\}$ is an abelian group under multiplication modulo 91. Determine the isomorphism class of this group.

   (b) Determine the number of isomorphism classes of abelian groups of order 400.

4. Do either of the following.

   (a) The relation $<$ on the natural numbers $\mathbf{N}$ can be defined by the rule $a < b$ if there is an $n \in \mathbf{N}$ where $a + n = b$.Assume the elementary properties of addition have all been proved and use the Peano Axioms to prove if $a < b$ then $a + n < b + n$ for all $n \in \mathbf{N}$.

   (b) Use the Peano Axioms to prove that the relation $<$ is transitive.

5. Do either of the following.

   (a) Describe the ring obtained from F_2 (the field $\mathbf{Z}_2$) by adjoining an element α satisfying $\alpha^2+\alpha+1 = 0$.

   (b) Describe the ring obtained from $\mathbf{Z}/12\mathbf{Z}$ by adjoining an inverse of 2.

6. Do either of the following [Recall that if F is a field then $F[x]$ is a principal Ideal Domain.]

   (a) Prove the ring $F_5[x] / (x^2 + x + 1)$ is a field. (Here F_5 is the field $\mathbf{Z}_5$.)

   (b) Prove the ring $F_3[x] / (x^3 + x + 1)$ is not a field (Here F_3 is the field $\mathbf{Z}_3$.)

7. Let a, b be elements in a field F with $a \neq 0$.Prove that a polynomial $f(x) \in F[x]$ is irreducible if and only if $f(ax + b)$ is irreducible.