Due January 26

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Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

"Mathematics is the language with which God has written the universe" -Galileo Galilei, physicist and astronomer (1564-1642)

Continuous Monotone Functions are Integrable

In this project you will make an argument that a function f that is both continuous and decreasing on an interval $[a, b]$ is integrable. This is essentially problem $78.b$ of section 5.3 of our textbook so be sure to read it and problem 77 as you work.

Suppose f is a continuous function on the interval $[a, b]$ and that f decreases throughout this interval.

1. Draw a graph of the function f on the interval $[a, b]$ that we will use for intuition in following your argument. Include a partition P of the interval into subintervals of **unequal** length.
2. Show that the difference between the upper and lower sums for f on this partition can be represented graphically as being smaller than or equal to the area of a rectangle R whose dimensions can be written in terms of $f(a), f(b)$ and $\Delta x_{\max}$ where $\Delta x_{\max} = \|P\|$. Specifically, show that

$$U - L \leq |f(b) - f(a)| \, \Delta x_{\max}$$

3. Use the above and the Sandwich Theorem to show that every Riemann sum of the function f on the interval $[a, b]$ limits, as $\|P\| \to 0$, to the same number and hence that the function f is integrable on the interval $[a, b]$.