Mathematics 290 Exam 2 Spring 2007

February 20, 2007

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Name

Technology used: _______________

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**
- **When given a choice, be sure to specify which problem(s) you want graded.**

Do both of these computational problems

C.1. Cast out vectors in $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -5 \end{bmatrix} \right\}$ to obtain a linearly independent set T of vectors where $\langle S \rangle = \langle T \rangle$.

C.2. If $\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\vec{u}_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, find a vector $\vec{u}_3$ for which $S = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ spans $\mathbb{C}^3$. Justify your answer.

Do any three (3) of these problems from the text, homework, or class.

You may NOT just cite a theorem or result in the text. You must prove these results.

M.1. Let $S = \{\vec{v}_1, \vec{v}_2\}$ be a set of non-zero vectors. Prove that S is linearly dependent, **if and only if**, one of the vectors in S is a scalar multiple of the other.

M.2. Suppose $S = \{\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_p\}$ is a linearly independent set and that $\mathbf{v} \notin < S >$. Prove the set $W = \{\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_p, \mathbf{v}\}$ is a linearly independent set.

M.3. Prove Theorem TMSM, Transpose and Matrix Scalar Multiplication

Suppose that $\alpha \in \mathbb{C}$ and A is an $m \times n$ matrix. Then $(\alpha A)^t = \alpha A^t$.

M.4. Suppose $S = \{\overrightarrow{v}_1, \overrightarrow{v}_2, \overrightarrow{v}_3, \vec{v}_4\}$ is a linearly independent set in $\mathbb{C}^{21}$. Is the set of three vectors $T = \{2\overrightarrow{v}_1 + \overrightarrow{v}_2 + 3\overrightarrow{v}_3 + \vec{v}_4,\ \overrightarrow{v}_2 + 6\overrightarrow{v}_3,\ 3\overrightarrow{v}_1 + \overrightarrow{v}_2 + 2\overrightarrow{v}_3 - 5\vec{v}_4\ \}$ linearly dependent or linearly independent? Justify your answer

Do one (1) of these problems you've not seen before.

T.1. Our author (Beezer) proved in one of the textbook exercises that if If $\vec{u}_1$ and $\vec{u}_2$ are both in $\langle S \rangle$, the span of S, then so is the sum $\vec{u}_1 + \vec{u}_2$. Use Beezer's result and the Principle of Mathematical Induction to prove that if $\vec{u}_1, \vec{u}_2, \vec{u}_3, \cdots, \vec{u}_n$ are all in $\langle S \rangle$ then so is the sum $\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \cdots + \vec{u}_n$.

T.2. Do both of the following.

(a) Prove that if $\vec{u}$ is a vector in $\langle S\rangle$ and α is a scalar, then $\alpha\vec{u}$ is also in $\langle S\rangle$. [Note that although $\vec{u}$ is a vector in the span of S it need **not** be one of the vectors in S.]

(b) Use anything we have studied (including part a. and the result of the Mathematical Induction problem) to prove the following.

If each vector in the set $T = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \cdots, \vec{v}_n\}$ is in $\langle S\rangle$, the span of S, then every vector in $\langle T\rangle$, the span of T, is also in $\langle S\rangle$.

T.3. Suppose that $S = \{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ is a set of vectors in $\mathbb{C}^{21}$ and that $T = \{\vec{w}_1 + \vec{w}_2 + \vec{w}_3, 2\vec{w}_1 + 3\vec{w}_2 + 4\vec{w}_3, \vec{w}_2\}$.[N T has exactly three vectors.] Prove that $\langle S\rangle = \langle T\rangle$. That is, prove that the span of S is the same set as the span of T.