

February 6, 2007

 Name

Technology used: _____

- – **Only write on one side of each page.**
- Show all of your work. Calculators may be used for numerical calculations and answer checking only.
- Be sure to include in-line citations every time you use technology and Include a careful sketch of any graph obtained by technology in solving a problem.

Do any six (6) of the following problems1. (10 points) Do **one** (1) of the following:

- (a) Use the definition of limit (which means you will have ε and δ in your answer) to prove that $\lim_{x \rightarrow 3} (5x - 8) = 7$.
- (b) Use the graphical interpretation of limits (using vertical and horizontal windows) to show that if $f(x) = \begin{cases} x + 1, & \text{if } x \leq 1 \\ x - 1, & \text{if } x > 1 \end{cases}$ then $\lim_{x \rightarrow 1} f(x)$ does not exist. Give details.

2. (8, 7 points) Without using a calculator, evaluate **both** of the following indefinite integrals

- (a) $\int \left(5 \cos(x) - \frac{2}{\sqrt{1-x^2}} + \sec(x) \tan(x) + \frac{1}{x^{2/3}} \right) dx$
- (b) $\int \frac{6+2\sqrt{t}}{t} dt$

3. (10 points) Let P be a partition of the interval $[0, 3]$ into n subintervals of equal length. Use sigma notation to write the lower sum L for the function $f(x) = 4 - x^3$ using this partition. **Do not simplify.** [Remember that the lower sum is the Riemann sum that is less than or equal to every other Riemann sum.]

4. (15 points) Without using a calculator, do **one** (1) of the following:

- (a) Find the derivative $H'(x)$ of

$$H(x) = \int_1^{e^{3x}} \frac{\ln(t)}{t^5 + 3} dt.$$

- (b) Find a function f that satisfies the equation: $\tan(x) = \int_2^x \sqrt{7 + f(t)} dt$.

5. (10 points each) Use substitution to evaluate **both** of the following

- (a)

$$\int \frac{\sin(\sqrt{x}) \cos^7(\sqrt{x})}{\sqrt{x}} dx$$

(b)

$$\int \frac{x (\arctan(x^2))^5}{1+x^4} dx$$

6. (15 points each) Do **both** of the following.

- (a) The base of a solid sits on the region in the xy -plane bounded by the x -axis and the graph of the semicircle $y = \sqrt{9-x^2}$. If cross sections perpendicular to the x -axis are rectangles with height twice as great as their base, what is the volume of the solid?
- (b) Find the volume of the solid obtained when the region in the first quadrant bounded by the curve $x = y - y^3$, the line $x = 1$ and the line $y = 1$ is rotated about the line $y = 1$.